

The Logical Atomism
of
Bertrand Russell

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INTRODUCTION.

Recent years have witnessed an increased interest in logical theory. One of the most interesting of the various turns taken by this investigation, is that which concerns itself with the problem of language as the instrument of thought. The approaches to this problem are many and various: linguistic, grammatical, psychological, logical and metaphysical. Of these possible approaches Russell has attempted only one—the logical. In a series of articles for the *Monist* 1918-19 entitled “The Philosophy of Logical Atomism,” he has turned his attention to precisely this question, and it is these articles, together with related references in his other works, which will afford the basis for the discussion which follows.

We have nowhere a more precise statement of the problem which he is considering than that he is “endeavoring to formulate a metaphysics upon the basis of a logic.” In general this is a sufficiently accurate statement of the problem. But it is important to realize the precise method according to which this “formulation” is carried out. The obvious starting-point would be thought itself, from its logical side. But since all thought takes verbal form in language, and there is a known relation between the form of thought and the form of language, Russell considers it simpler and more convenient to start with thought as it is symbolized, i.e. with the proposition. Hence the initial study is with the proposition as a form of words. On the verbal side every proposition has a certain form (it is a succession of words) and position within this form is essential in the determination of the meaning. The first step beyond the verbal form is the grammatical form; this is the initial process in interpretation. Grammatically every proposition consists of two parts, called respectively the subject and the predicate. The second step is the passage from the grammatical form to the logical form. On this level there are three questions. (1) What typical logical forms of propositions are there? (2) If these are reducible to one another, how is the reduction carried out? (3) If they are irreducible, in what does the ultimate difference consist? At this stage we are on the level of thought, for the logical meaning of a proposition is what is thought about that proposition. Thus from language we have passed to thought. But thought itself is a picture of the world, and there is a known relation between the form of thought and the form of fact. Consequently the next step is the passage from “thought” to “things,” for the form of the latter must be derivable from the form of the former. Thus upon the basis of an analysis of propositions as verbal forms, we are enabled to ascertain the nature of the world.

Obviously the critical point in such a procedure lies in the validity of the assumption as to the identity of form exhibiting itself between language and the world. For if such an

assumption can be shown to be false, it casts doubt upon all of Russell's metaphysical conclusions insofar as they are dependent upon a logical atomism. The assumption, stated as explicitly as possible, is that "there is an objective complexity in the world, and it is mirrored by the complexity of propositions."¹ The use of this assumption lies in passing from the "form of a proposition" expressing a fact, to the "form of the fact" which is thus expressed. The dangers in such a passage from a proposition to its "meaning" are incomparably enhanced because there are "a strictly infinite number of different things that 'meaning' may mean,"² i.e. facts "mirror" propositions in an infinite number of ways. The assumption, therefore, is not merely in the fact of the mirroring, but in the fact that there are different types of mirroring.

This, then, affords a crucial point in the criticism of Russell's system, and we shall be concerned with it throughout our discussion. It is one aspect of the general problem of "symbolism." In how far does the form of the proposition correspond to the form of the fact? If they are closely similar, we may be safe in concluding something about the word simply by an analysis of propositions. If they are partially similar, and we know in what aspects they are similar, we may still claim knowledge of the world on the basis of logical knowledge. On the other hand, if they are similar only in accidental aspects, i.e. if we know no law of correspondence of forms, or if we know definitely that there is no correspondence at all, our analysis of propositions will afford no basis for concluding anything about the nature of the world. It is with this question that we shall be concerned in Chapters Three and Four. It becomes identified, strictly speaking, with the theory of truth and falsehood.

But the general problem of symbolism appears in the small in the analysis of propositions into terms, and the way in which terms "mean" what they do mean. Proper names mean in one way, for example; common nouns in another, and relational words in still another. Hence we have a separate problem with regard to each. These will be dealt with in Chapters Two and Four.

The primary claim to originality in my treatment lies in the arrangement of the whole. Russell's failure to formulate precisely the problem of "meaning" has prevented him from making certain necessary distinctions, and the result has been an obscuring of the whole problem. I hope to avoid this confusion by distinguishing the proposition as a verbal form, from the proposition as a meaning, and both of these from the objective reference of a proposition. The chapter on "The Proposition as a Complex" is a sincere attempt to bring all of

¹L. A. 520. See page 62 for table of abbreviations.

²*Ibid* 506.

Russell's isolated and random statements upon the subject of word-meaning into a consistent whole. He has nowhere treated this problem by itself, consequently the results of this chapter are largely conjectural. Random criticisms will be made throughout the discussion. In particular I shall attempt to show the impossibility of *defining* existence in logical terms, i.e. passing to a definition of existence on the basis of the form of thought. In the final chapter I shall present some reasons in favor of accepting molecular facts, and of denying general and particular facts. The conclusion will indicate what must be accepted and what must be rejected with regard to the general problem of logical atomism, and the direction in which we must look for the complete solution. Throughout the discussion the metaphysical position expressed by Russell in his earlier works is assumed. It thus attempts to catch him in his neo-realistic phase, before his swing over to neutral monism.

One point further needs to be suggested. As Wittgenstein has pointed out, any discussion of logical atomism is a *petitio*. It claims to be a presentation of the fact of "symbolism" as applied to propositions. It maintains, that is, that every proposition is a "picture" of a fact. But if every proposition is a picture of a fact, then every proposition contained in such a discussion must be either directly or indirectly a picture of the fact of symbolism. Consequently the whole treatment of the problem of symbolism assumes the fact of symbolism itself, and is therefore a clear case of begging the question. "Whereof one cannot speak, thereof one must be silent."

CHAPTER ONE

THE PROPOSITION.

Since we are to determine the structure of the world upon the basis of the form of thought, our investigation must begin with the unit of thought. This is, of course, the proposition. Russell's definitions of this term are various. It is the unit of logical thought in the sense that it "is going to be our typical vehicle on the duality of truth and falsehood."³ A proposition is "a form of words which must be either true or false."⁴ It "is a series of words(or sometimes a single word) expressing the kind of thing that can be asserted or denied."⁵ The strictly logical definition is in terms of implication; since every proposition implies itself, and only a proposition implies itself, we may take " p implies p " as the definition of the proposition " p ." This definition, however, need not concern us here.

It seems, then, that Russell is in essential agreement with traditional logicians at least in the definition of proposition, however greatly he may choose to depart from their views in what follows. Yet certain random suggestions which he makes, intimate that the term "proposition" is not to be limited merely to a form of words expressing truth or falsehood. These suggestions amount, at times, to direct contradiction of the views we have stated. In his *Introduction to Mathematical Philosophy*, for example, he chooses to define proposition as "primarily a form of words" in order not "to exclude other than verbal symbols, or even mere thoughts if they have a symbolic character."⁶ The term "proposition" may be used to designate any symbol which may be true or false. The content of an image-belief, for example, may be a proposition.⁷ In his earlier works the view is emphasized even more strongly: "A proposition, unless it happens to be linguistic, does not itself contain words," and he speaks of a confusion which arises from accepting "the notion that *words* occur in propositions."⁸ The contradiction between this view and the position we first stated, is clearly indicated if we take his definition of a proposition as "a sentence in the indicative, a sentence asserting something, not questioning or commanding or wishing."⁹

How are we to reconcile these two views? They seem to me to indicate a shifting use of words rather than a direct contradiction. A proposition in any event must be something; a symbol must be an entity of some kind. On the other hand a symbol, and derivatively a proposition, must stand for, or represent, or "mean" something else. Every symbol exists,

³L.A. 504.

⁴S.M. 52.

⁵A.M. 241.

⁶Page 155.

⁷A.M. 241.

⁸P. of M. 47.

⁹L.A. 504.

therefore, to use Bradley's distinction (which he applies, however, primarily to ideas) both as a content and as a meaning. The proposition "Socrates is mortal" may be considered from the physical side as a series of written symbols, the "is" following "Socrates" and preceding "mortal," or as a succession of noises with the "is-noise" following the "Socrates-noise" and preceding the "mortal-noise." This is the proposition in its verbal aspect. From the meaning side, however, the proposition affirms that the individual Socrates is related in a manner called the "epsilon relation" to the class mortal. It is of no particular consequence, therefore, which term of the symbolical relation we choose to designate by the word "proposition," so long as we remember that the two terms are distinct. But to use the word, now for one term, and now for the other, as Russell apparently does, is both a source of confusion to the reader and a source of danger to the one who makes such use. I shall try to avoid this confusion and danger in what follows by using the term "proposition-verbal" to designate the symbol from its physical side. When I wish to express the symbol on the side of its meaning, I shall speak of the "meaning of the proposition" or more briefly "propositional-meaning." The word "proposition" will be a generic term to include both proposition-verbal and propositional-meaning, and will be used in this sense when no possible confusion can arise. It is, of course, the propositional-meaning and not the proposition-verbal which is either true or false, that is, it is the meaning of the proposition which "points toward or away from the fact." That which is symbolized by a propositional-meaning, i.e. the fact, will be designated by the term "objective-reference." For lack of space, what Russell terms "image-proposition" will not be considered in our discussion.

It might seem that this is an artificial distinction. When, one is inclined to ask, are we ever concerned with the proposition-verbal as distinct from its meaning? Isn't our interest always in the proposition as a symbol? The answer is, Usually, but not always. And one time when we are interested in a proposition-verbal is in a logical atomism. For "the meaning of a proposition is derivative from the meanings of its constituent words."¹⁰ In the atomic proposition, for example, the meaning of the proposition is obtained by substituting for each word what it means. The relation of the meaning of the proposition to the proposition-verbal is thus very simple. On the other hand when we come to general and particular propositions, the meaning is derived from the verbal form only by a complicated analysis. But the fact that the meaning of the proposition is always derivable (at least theoretically) from the proposition as a form of words, necessitates separate terms to designate the beginning and end of this derivation.

¹⁰A.M. 273.

It is necessary to say something further about symbolism. A proposition is a symbol; that is, every proposition exhibits itself in some sort of meaning. A symbol is defined as "anything that means something else."¹¹ All language is therefore a symbol, every word is a symbol, every sentence is a symbol. Yet it must not be supposed that there is only one relation which is termed symbolic. For "there are different kinds of symbols, different kinds of relation between symbol and what is symbolized."¹² Terms do not "mean" in the same way as propositions do, and among terms, proper names "mean" in a different manner from general names, and both differ from relational words. Among propositions, moreover, atomic propositions do not symbolize in the same way as molecular, and neither of these "means" in the same manner as a general proposition. Thus the problem of symbolism carries a great deal of significance and resolves itself into a series of separate problems.

With regard to propositions the problem of symbolism exhibits itself in two questions, which if not distinguished, lead to all sorts of confusion. There is first of all the question as to the relation between the proposition-verbal and its meaning. What, in other words, is that "functional relation" which the meaning of the proposition bears to the meaning of the words? This divides into two further questions: (1) How do words mean? (2) How is the meaning of the proposition as a whole derived from the meanings of its verbal constituents? The answer to the first will be attempted in the chapter on "Proposition as Complex;" the answer to the second will be discussed in the chapter on "Proposition as Unit." In the second place, then, there is the general question as to the relation between the meaning of the proposition and the objective fact which makes it true or false. Every proposition is true or false by virtue of something outside itself,¹³ and it becomes a problem to determine the nature of this relation. This will be considered in the chapter on "Proposition as Symbol."

One of the greatest difficulties in the understanding of Russell's logical atomism is his failure to sharply distinguish these two aspects of the problem of symbolism. When he uses the word "proposition" one can seldom be sure whether he is referring to the proposition as a form of words, to the meaning of the proposition, or to the objective fact by virtue of which

¹¹L.A. 506.

¹²*Ibid* 505.

¹³Russell apparently takes this as one of his "undeniable data." He does criticize the coherence theory of truth, however, and concludes that it is unacceptable. This criticism occurs in an essay "The Monistic Theory of Truth" in *Philosophical Essays*, and in random places throughout his writings.

it is true or false.¹⁴ For example, he analyzes the symbolic problem of language as follows: "First, there is the problem what actually occurs in our minds when we use language with the intention of meaning something by it; this problem belongs to psychology. Secondly, there is the problem as to what is the relation subsisting between thoughts, words or sentences, and that which they refer to or mean; this problem belongs to epistemology. Thirdly, there is the problem of using sentences so as to convey truth rather than falsehood; this belongs to the special sciences dealing with the subject-matter in question. Fourthly, there is the question: what relation must one fact (such as a sentence) have to another in order to be capable of being a symbol for that other."¹⁵ I am not concerned with criticizing the last two problems. The third refers to the problem of truth and error as it pertains to any given body of knowledge; the fourth is the problem of symbolism of language in its general form. But there is an ambiguity in the first problem. If Russell means by "what actually occurs in our minds" the images or ideas in the Berkeleyan sense which are present in our minds as events, then it is a psychological problem. But if he intends to imply that what is thought about a proposition, i.e. what is meant by a proposition-verbal, is a psychological problem,¹⁶ then I think he has misconceived it. The problem of meaning is not, strictly speaking, a psychological problem; it is a problem for logic, or perhaps metaphysics. In this sense logic concerns itself with thought. But it considers thought in its universal aspect, not as an event in one mind. Consequently there is a necessity about logic; if the relation of symbol to what is symbolized is unique in any given situation, one can mean only one thing whenever such a symbol is employed. Hence there is a corresponding ambiguity in the second problem of symbolism as formulated by Russell. There is not one problem as to the "relation subsisting between thoughts, words or sentences and that which they refer to or mean." In the first place "thoughts" must be distinguished from "words and sentences," they are on different levels and

¹⁴An exception to this is Russell's article on "How Propositions Mean," *Aristotelian Society* Pub. v. 2, p. 29, when he sharply distinguishes between "word-proposition," "image-proposition" and "objective-reference."

¹⁵*Tractatus Logico-Philosophicus*, Intro. pp. 7 and 8.

¹⁶The authors of *The Meaning of Meaning* (C. K. Ogden and I. A. Richards) tend to consider the problem of meaning in this way, that is, as a psychological problem. Consequently when they have solved the problem of belief (chap. iii) they claim to have solved the general problem of truth and falsehood. This is not the case . . . The diagram given on page 14 is suggestive as a starting-point for the solution of the problem of meaning. If the vertex of the triangle, i.e. the thought or reference is considered on its logical and not its psychological side, the diagram corresponds to the analysis which I have given. The authors fail to realize, however, that every attempt to picture the problem of symbolism is a *petitio*.

the problem of symbolism is different in each case. In the second place "words" must be distinguished from "sentences" for certain words symbolize entirely outside of the context of a proposition, but no proposition has meaning apart from the meanings of its constituent words.

It becomes possible, therefore, to profit by Russell's confusion, and to state the problem more precisely, using a less ambiguous terminology. There are two general problems, with a third whose solution is dependent upon these two. (1) How does language mean, i.e. what sort of relation subsists between language and thought (in its universal aspect)? The converse of this problem is that of determining how "thought" takes verbal form. The two sub-problems are (a) How do words mean? (b) How do propositions-verbal mean? (2) How do thoughts, i.e. meanings, refer to facts or particulars or universals? This divides into the problems: (a) How do propositional-meanings refer to facts? (b) How do word-meanings refer to particulars and universals? The derivative problem is this: (3) How does language refer to facts or simples? The relation of language to fact is the relative-product of the relations language-to-thought and thought-to-fact. Each of these problems, however, is a distinct one, and it is essential to a clear understanding of the subject that they be not confused.

It is strange that Russell should have so confused the problem in the introduction to a book where the distinction is made so clearly. Wittgenstein leaves no room for ambiguity. 2.1: "We make to ourselves pictures of facts." 3.0 "The logical picture of facts is the thought." 3.1: "In the proposition, the thought is expressed perceptibly through the senses." 3.12: "The sign through which we express the thought, I call the propositional sign." 4.0: "The thought is the significant proposition." 4.001: "The totality of propositions is the language." There is no equivocation here. The "propositional sign" is what I have designated by "meaning of the proposition," i.e. "the proposition sign in its projective relation to the world" (3.12). "Thought" may be considered from its two sides: On the one hand it is a "picture of the world," i.e. it refers to facts. On the other hand it is a "significant proposition," i.e. the "applied propositional sign" (3.5). It is just this double symbolism between a proposition-verbal and a fact which permits us to distinguish between meaning and truth. A proposition-verbal may have meaning even though false; if every proposition-verbal meant a fact, this would be impossible. We shall consider this more in detail later.

One more idea, which is really of fundamental importance in Russell's system and a necessary condition to any understanding of it, must be introduced at this point. I refer to the notion of propositional-function.

What is a propositional-function?¹⁷ "A propositional-function is simply any expression containing an undetermined constituent, or several undetermined constituents, and becoming a proposition as soon as the undetermined constituents are determined."¹⁸ A propositional-function thus asserts nothing and therefore cannot be a proposition; it is neither true nor false by itself, and consequently cannot be a proposition. It is, as Russell suggests,¹⁹ nothing, but that does not diminish its importance. A propositional-function is best described as a sheaf of propositions represented in a single symbol; it is an expression indicating the form of a number of propositions. Examples of propositional-functions are, "x is a man," " $(x+y)^2=x^2+2xy+y^2$," "If x is a man, then x is mortal," "x is a centaur," etc. None of these expressions can be said to make an assertion, and therefore to be either true or false until something is said about the range of values of the undetermined variable. "x is a man," for example, will be true for certain x's, namely those which are men; the function, "If x is a man, then x is mortal" is true for all x's, those which are men and those which are not; the function "x is a centaur" is true for no x's. A propositional-function with the range of the variable determined is a proposition. Every propositional-function, therefore, must be modified by a clause asserting that it is true for all, some or no values of the variable. Conversely every proposition containing the words "all," "some," "no," "a," "any," etc., can be resolved into a propositional-function. This brief treatment will suffice here.

¹⁷Russell's best presentation of this notion is contained in his *Introduction to Mathematical Philosophy*, chap. xv.

¹⁸L.A. 192.

¹⁹*Ibid.* 193.

CHAPTER TWO

THE PROPOSITION AS A COMPLEX

Every proposition is complex; it consists of words. The words are the elements of the complex. As to what is meant by complexity, we need not here detain ourselves. Russell defines it as "anything that occurs in the universe and is not simple."²⁰ We may assume that anything which permits of analysis into simples is complex. But a proposition-verbal is more than this; if it were only a complex we should have no index as to how far the analysis should go. We might, for example, analyze the proposition-verbal into letters and suppose that these were the simples of which the complex were composed. We are prevented from doing this because of the fact that a proposition-verbal is not only a complex; it is a complex *symbol*.²¹ "A symbol may be defined as complex when it has parts that are symbols. In a sentence containing several words, the several words are each symbols, and the sentence composing them is therefore a complex symbol in that sense."²² The letters of which a proposition-verbal consists would not be simples, for they are not themselves symbols.

There is consequently one step which is preliminary to the determination of the manner in which propositions-verbal symbolize. We must consider how words in isolation mean, for "whatever may be the meaning of the complex symbol, it is clear that it depends upon the meanings of the separate words. Thus before we can hope to understand the meaning of a proposition as a form of words, we must understand what constitutes the meaning of the single words."²³

This might seem to be a very simple problem. We have only to take any random group of words, determine how they mean, and thus consider our problem solved. But two difficulties arise: (1) Words mean in a number of different ways. (2) The way in which a given word means is dependent,²⁴ in general, upon its function in the sentence. The escape from the first difficulty lies in the fact that there are certain general types or classes of words, all the members of which mean in a similar manner. Thus it becomes a problem simply of determining what are the general types, and how each is characterized with regard to the nature of meaning. The second difficulty, in view of what we have seen above, formulates itself in the nature of a paradox. The meaning of the proposition is dependent upon the meanings of the several words, yet in many cases the meanings of the words are derived from their

²⁰P.M. 47. In another place (L.A. 519) Russell intimates that complexity is indefinable; it is simply apprehended.

²¹H.P.M. 7.

²²L.A. 504.

²³H.P.M. 7.

²⁴Except in the case of proper names. See pp. 16-18.

function in the sentence. The solution of the paradox lies, as we shall see, in the discovery of a reciprocal relation which propositions-verbal and words have to one another. The meaning of a proposition is dependent upon two factors—its form and its content—which are indissolubly connected. Every proposition must have a form, yet a form must always be of something. Word-meanings, consequently, which are elements of the complex propositional-meaning, can be adequately considered only as *vital parts* of the complex. The complex, on the other hand, is meaningful only when made up of elements. Until we are in a position to justify this view more in detail, we shall incorporate the second of these difficulties into our procedure as a definite, conscious assumption. An assumption which is clearly recognized and frankly stated cannot be objected to. In most cases we shall not find it possible to forget the assumption, for it is continually being thrown back at us in the course of the analysis. This chapter will attempt to justify such an assumption.

The grouping of words into types is really a matter for grammar, though it has its reflection in logic, and as a result, in metaphysics. Every sentence when completely expressed has a subject and a predicate; there is always an assertion about something. Thus grammar distinguishes within every sentence the subject, or that about which something is asserted, and the predicate, or that which is asserted about the subject. These might be called the elements of the sentence. On the side of logic, corresponding to the elements of the sentence, we have terms of propositional-meaning. But to conclude, as traditional logic has done, that the grammatical analysis into subject and predicate is an ultimate analysis, is to commit a gross error; for it suggests that from the point of view of logic there are only two sorts of terms, substantives and attributes. Logicians are accustomed to defining term as anything that may be the subject or predicate of a proposition.²⁵ But a closer analysis of predicates will reveal the fact that they are of two kinds which are fundamentally different; on the one hand there are those which attribute something to the subject; on the other there are those which assert that the subject is related to something. Consequently logic must admit as a third type of term, irreducible to the others, that which we may call a relational term. The admission of this type of term has profound effect upon one's metaphysics.²⁶

It is this fact, I take it, which has led Russell to attempt a definition of term which is logical rather than grammatical. As a result we have term defined as "whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*."²⁷ One feels that this is a very

²⁵H.W.B. Joseph: *An Introduction to Logic*, chap. 2.

²⁶A more complete discussion of this point will be found in chapter iii.

²⁷P. of M. 43.

unsatisfactory definition until he is reminded that term is the "widest word in the philosophical vocabulary," and is used synonymously with such words as "unit, individual and entity." "A man, a moment, a number, a class, a relation, a chimera, or anything else that can be mentioned, is sure to be a term."²³ Thus the word can have no significance in use until it is subdivided and one is told what *sorts* of terms there may be.

When we come to the discussion of types of terms, we can clearly see that Russell has fallen back upon the grammatical distinction. For "terms may be distinguished as of two kinds, things and concepts. The former are the terms indicated by proper names, the latter those indicated by all other words."²⁹ But he does not *stop* at the grammatical distinction. For among concepts "two kinds at least must be distinguished, namely those indicated by adjectives and those indicated by verbs. The former kind will often be called predicates or class-concepts; the latter are always, or almost always, relations."³⁰ In a later article³¹ Russell rejects this use of the words "things" and "concepts" to express the two general kinds of terms. In order to avoid the controversy as to whether verbs are or are not reducible to predicates, he expresses the distinction as an "opposition which puts predicates and relations on one side, and everything else on the other."³² The determining force of the grammatical distinction upon the logical, is clearly evident. Russell's variation from the traditional logic is in his having carried the analysis one step further.

The problem of determining how words mean, then, will first be considered in its simplest aspect, i.e. where there is a one-one correspondence between the word and the term. There will be three problems here: 1. How do proper names mean? 2. How do adjectives and general names mean? 3. How do verbs mean? We shall take these in order. Having considered them, we must turn to certain less simple cases, namely descriptive phrases, where a group of words means (in this case incorrectly) a single term.³³

1. Russell's use of the term "proper name" must not be confused with the accepted every day use, although it has a meaning which is derived therefrom. He has used it in a strict logical sense. A proper name usually means a particular individual or object; it is essential to a proper name that it

²⁸*Ibid.*

²⁹P. of M. 44.

³⁰*Ibid.* 44.

³¹Aristotelian Society Pub., "On the Relation of Universals and Particulars." 1911-12, p. 4.

³²*Ibid.* p. 22.

³³In all of Russell's recent discussions of the problem of word-meaning, he tends to over-emphasize the "understanding" side of meaning. (A.M. chap x.) Thus the problem of the meaning of a word becomes the problem of its use. In all of these discussions, Russell leans noticeably toward behaviorism.

be unambiguous and point to only one object. It is in this sense that Russell's use of the term is derivative. Logically also, a proper name is a word which points, and there is no ambiguity in the pointing.

But Russell's use is more confined. A proper name always means one of the "ultimate indivisible constituents of the world." A study of Russell's theory of the external world reveals the fact that the world is constructed out of *sensibilia*. In his logical atomism he calls these "particulars,"³⁴ and he defines proper names as "words for particulars."³⁵ Furthermore a proper name can only be used validly when it means an actual object of sense—a "particular with which one is acquainted at the moment."³⁶ Let us see what characteristics proper names have as a result of these designations.

(a). Proper names "mean" in a way in which nothing else means. The meaning of a proper name is always something actually present. There is no intermediary between the word as a physical event, and the actual object designated by the word; the meaning of the word is the object. One cannot use such a word unless he is experiencing the sense-datum. Consequently a proper name means something different from one moment to the next (i.e. as the sense-datum changes) and never means the same to speaker and hearer (because of the difference of point of view.)

(b). Proper names must refer to the simple. The object denoted by a proper name can never be broken up into parts; if a supposedly simple object can be broken up into elements, it is no longer capable of being named. If the parts are simples, then these may be named. Hence, granting Russell's metaphysics, a proper name can never refer to an "object." A table, for example, is a class of appearances, and is therefore not simple. A proper name cannot even refer to a single appearance of the table, for that is a complex consisting visually of light and dark patches, shapes and sizes, colors, etc., having spatial relations to one another. A proper name does not name a complex; a complex cannot be named.

(c). Proper names are the only strictly non-connotative names possible. For examples of proper names, one is limited to the words "this" and "that." But these do not describe, they simply point. *Sensibilia* cannot be described, hence they must be tagged. The words "this" and "that" as strictly

³⁴In one place (M.L. 213) Russell defines particulars so as to "include existents and complexes of which one or more constituents are existents, such as 'this-before-that,' 'this-above-that,' 'the-yellowness-of-this'." This is in direct contradiction with his use of the word elsewhere. (See L.A. 523.) Such complexes as "this-before-that," etc., are here spoken of as facts, and are on an entirely different level from that occupied by particulars. This is one of the many examples of Russell's equivocal terminology.

³⁵L.A. 523.

³⁶*Ibid* 524.

logical proper names, simply denote; they cannot connote for there is nothing to connote—a simple does not have a quality.

Although there are quite likely such things as proper names, it seems possible to criticize Russell's interpretation of this notion. Even though one admit his metaphysical system, it is apparent that proper names are not "atoms of logical analysis,"³⁷ as he claims, but atoms of metaphysical analysis. This conception of proper names is clearly a construction of Russell's to meet the demands of his metaphysical system. He has analyzed the world and found it to be made entirely of sense-data—particulars—elements of the world, each of which "stands alone and is completely self-subsistent."³⁸ These particulars cannot be described; in order to understand a particular one must be acquainted with it, i.e. actually experience it in sense. Now it becomes necessary to talk about particulars. Since they cannot be described, they must be tagged; the tags of particulars are proper names. But this is not constructing a metaphysics on the basis of logic, as Russell claims to be doing;³⁹ it is quite definitely creating a logic on the basis of a metaphysic.

2. The second problem is not so easily disposed of. It is really a compound problem: How do general names and how do adjectives mean? Summarily stated, the solution seems to be the following.⁴⁰

Just as proper names mean particulars, so general names and adjectives mean "universals";⁴¹ the relations in the two cases are analagous but not identical. Particulars are ultimates of the world, and the relation of proper name to particular

³⁷L.A. 497.

³⁸L. A. 525.

³⁹L.A. 496.

⁴⁰Due to the fact that Russell does not directly concern himself with this particular problem, the analysis which follows is largely conjectural as far as Russell's actual system is concerned. It would be difficult to say to how much of it he would assent. I have attempted to show how we should expect Russell to have solved the problem, in the light of certain meager suggestions which his works offer. In particular Russell has remained strangely silent upon the problem of the nature and status of universals. Since his *Problems of Philosophy* there has been practically nothing. Whenever he has chosen to express himself, it has been in a manner which implies that he has greatly modified his former view. His earlier works suggest that he admits as part of the world, an actual entity—a universal—of which we are immediately aware. (M.L. 212; P. of P. 158.) But later (A.M. 228) he shifts his position and affirms that "universals are part of the structure of the world, but they are an inferred part, not a part of our data." Elsewhere in the same work he openly expresses skepticism about the whole matter. "Whether there is a universal called 'whiteness' . . . is a question . . . which I believe to be strictly insoluble." (196.) His definition of "white" as a "class of particulars" suggests that there is no universal white, but only a class of "whites." I have tried to combine all of these positions into a view which, although it admits universals as stages in meaning, gives them only a hypothetical existence in the actual structure of the external world.

⁴¹P. of P. 145.

can be called "naming." Universals, however, if we are to accept Russell's later view cannot be proven to be an actual part of the make-up of the world. Consequently they cannot be the ultimate reference of general names and adjectives, and the relation cannot be called naming. There is an intermediary which enters into the meaning relation at this point and destroys its immediacy. This is the concept. General names and adjectives denote concepts, "awareness of universals is called *conceiving*, and a universal of which we are aware is called a concept."⁴² A concept is therefore a universal which is "meant" by someone, who employs as the symbol of the meaning either a general name or an adjective.

Although concepts are of two kinds—those symbolized by verbs and those symbolized by predicates—it is only the latter with which we are concerned at this point,⁴³ for concepts indicated by general names and adjectives are predicative. But "concepts which are predicates⁴⁴ might also be called class-concepts, because they give rise to classes."⁴⁵ We are thus plunged headlong into the notion of class. A general name such as "man," for example, "means a whole class of such collections of particulars as have proper names. The several members of the class are assembled together in virtue of some similarity or common property."⁴⁶ Adjectives are defined after the same pattern; "we may take the word 'white' as denoting a certain set of similar particulars or collections of particulars."⁴⁷

We are apparently at a standstill until we have determined the answers to two questions: (a) What is meant by a class? (b) Is the notion of class ultimate, or must we look further for that which general names and adjectives mean?

⁴²M.L. 212.

⁴³Concepts which occur as subjects (in the logical sense of the word, i.e. as terms of a relation) can be reduced, if we properly analyze the proposition in which they occur, to predicates. This is, at least, Russell's later view. (L.A. 34-5) "Man is mortal," for example, is reducible to an implicative relation between propositional-functions, in which "man" and "mortal" are resolved into predicates. In view of Russell's denial of the necessity for assuming universals we can no longer accept the analysis given in the earlier work (M.L. 212) as final.

⁴⁴A great deal of confusion is caused by Russell's shifting use of the word "predicate." He uses it in at least four different ways, which are quite distinct. (a) To indicate the grammatical predicate of a proposition, i.e. what is asserted about the subject. (S.M.45.) (b) To indicate an attributive adjective. (P. of M. 56.) To include both attributive adjective and what is called in grammar a "predicate noun." (d) To designate the actual metaphysical existence which is the ultimate reference of a "predicate" in the third sense. (L.A. 509.) We shall use the word in the third and fourth senses, leaving it to the context to indicate whether we refer to the word or to the existent.

⁴⁵P. of M. 54.

⁴⁶A.M. 194.

⁴⁷*Ibid* 196.

Class is defined⁴⁸ as a "numerical conjunction of terms." We need not ask here whether this is a circular definition; the word "conjunction" at least suggests that it is. Nor need we raise the question as to whether every class need be numerical. These are irrelevant matters for a logical atomism. Other words for class are "collection," "set," "group," etc. Two points should be emphasized. (1) We must not identify classes with mere heaps or conglomerations because of the necessity for including among classes on the one hand the null-class, which is in no sense a heap, and on the other the unit-class, which is not identical with its sole member.⁴⁹ (2) A class may be defined in one of two ways, by extension or by intension,⁵⁰ of which the latter is more fundamental.⁵¹ The extensional definition can always be reduced to the intensional, but the converse is not true. This is the important characteristic of classes for our purpose. For we may consider a class as a group or collection of entities, membership in which is determined by possession of the attribute which is spoken of as defining the class. "A class (which is the same thing as a manifold or aggregate) is all the objects satisfying some propositional-function."⁵²

So much for the answer to the first question. Now, can we assert that the notion of class is ultimate, and may therefore be spoken of as the "meaning" of general names and adjectives, or must we carry on the analysis further in an effort to reduce this notion? The answer to this question involves the introduction of the idea of "incomplete symbol," which plays a large part in Russell's view. It consists of two ideas: (a) "By an 'incomplete' symbol we mean a symbol which is not supposed to have any meaning in isolation, but is only defined in certain contexts. . . ."⁵³ (b). Any proposition-verbal in which an incomplete symbol occurs, does not, in fact, in its meaning contain any constituent corresponding to that symbol; when the propositional-meaning is properly analyzed, the symbol will disappear.

The words for classes are incomplete symbols. In accordance with the above description, this will involve two points. In the first place, a symbol for a class will have no meaning in isolation; it is defined only in the context of a proposition-verbal. In the second place, there is no actual entity—"class"—which can be spoken of as the meaning of a general name or adjective. Let us consider each of these.

What is meant when we say that a word means a class,

⁴⁸P. of M. 67.

⁴⁹I.M.P. 183.

⁵⁰*Ibid.* 12.

⁵¹This flatly contradicts the view of P. of M. 69, where he states that extension and intension are logically on a par.

⁵²P.M. 24.

⁵³P.M. 69.

that the word "man," for example, means the class of men? A class, we have seen, is nothing more than the grouping of certain objects together. It is evident that we know nothing about a class until we know how the grouping is being carried out, i.e. why certain objects are included in, and certain others excluded from the class. The problem of the class, therefore, is the problem of determining membership in the class. "Red," we say, means a class? But what sort of a class? The class of red things. And what are red things? All those things which "are red." We must therefore determine what it is to "be red:" i.e. we must have some sort of instrument for designating the membership of our class. According to Russell, it makes no great difference whether we take "being red" as ultimate, or whether we define red by relation of similarity to a standard thing. The former would appear to be more simple. Consequently to understand "red," we must understand "being red," and the meaning of "red" is "being red." But this involves a proposition, i.e. "a thing is red." "When you understand 'red' it means that you understand propositions of the form that 'x is red'."⁵⁴ This is not true of proper names. A proper name means a particular directly, and to understand a proper name, one must only be acquainted with the particular; such a procedure does not involve any suggestion of a proposition. Predicates, however, mean only as elements of propositions; this is what is signified when we say they are symbolized by incomplete symbols. We have here the first corroboration of the fact that words in isolation mean nothing.⁵⁵

It should be apparent by this time what is involved in the second characteristic of a class as an incomplete symbol. There is no actual entity called a class, which enters into the propositional-meaning. "A class-concept is not itself a collection, but a property by which a collection is defined."⁵⁶ "Symbols for classes are mere conveniences, not representing objects called 'classes' classes are in fact logical fictions." They are not to "be regarded as part of the ultimate furniture of the world."⁵⁷ Since the extensional definition of class may always be reduced to the intensional, we have a key to what is meant by saying that a class is a logical fiction. A class may be replaced by its defining characteristic in the form of a propositional-function. The class of red things will be replaced by the propositional-function, "x is red." Membership in the class is determined by the range of values of the variable. If the class has members, then the propositional-function will be true for at least one value of the variable; if it has no members, it will be true for no values; if the class is the universe, then

⁵⁴L.A. 34.

⁵⁵See above p. 14.

⁵⁶P. of M. 115.

⁵⁷I.M.P. 182.

it will be true for all values. Thus classes are explained away and we can "avoid the need of assuming that there are classes."⁵⁸ General names and adjectives therefore mean only as elements of a proposition. They have no meaning in isolation and are as a result sharply distinguished from proper names. In other words, general names and adjectives are "relative" words, possessing meaning only in virtue of the proposition of which they are elements. This brings us to our consideration of verbs.

3. Just as concepts which are predicates give rise to classes, so concepts which are verbs are called relational-concepts. "Every verb, in the logical sense of the word, may be regarded as a relation."⁵⁹ Verbs as grammatically considered do not always symbolize the entire relation, they are supplemented sometimes by prepositions.⁶⁰ But "the verb gives as it were, the essential nerve of the relation."⁶¹

As a preliminary definition of relation, we may take Russell's statement that it is "a concept which occurs in a proposition in which there are two terms not occurring as concepts and in which the interchange of the two terms gives a different proposition."⁶² This is evidently a definition only of dyadic relations; there may be relations of any degree of complexity. Russell speaks by analogy of a monadic or one-termed relation;⁶³ this is the relation symbolized by a "predicate." All of our discussion of general names and adjectives, that is, might have been included under the heading of monadic relations. But it is well to remember that such a use of the term relation, can only be analogy. In general Russell makes a sharp distinction between subject-predicate propositions and relational propositions, even to the extent of asserting that predicates and relations have fundamentally different types of existence.⁶⁴ What we have to say about relations here will exclude monadic relations, and apply only to the higher types. Every relation in this sense implies at least two terms.

Relations may be defined either by extension or intension. Taken in extension, they are "logical fictions."⁶⁵ A relation taken in extension would be regarded "as a class of couples; i.e. the couple (x,y) is to be one of the class constituting the relation R if x has the relation of R to y."⁶⁶ A foot-note adds that "such a couple has *sense*, i.e. the couple (x,y) is different from the couple (y,x) unless $x=y$. We shall call it 'a couple with sense' to distinguish it from the class consisting of x

⁵⁸I.M.P. 184.

⁵⁹P. of M. 52.

⁶⁰A.M. 196.

⁶¹L.A. 523.

⁶²P. of M. 95.

⁶³L.A. 522.

⁶⁴L.A. 365.

⁶⁵L.A. 222; P. M. 85.

⁶⁶P.M. 27.

and y.” But classes themselves are logical fictions, and consequently to define relation as a class of a certain type is to reduce that also to a logical fiction. Hence if relations are to be taken in extension, the words which symbolize them will all be incomplete symbols. This implies that the word for a relation has no meaning apart from a proposition-verbal which contains it. To understand “before” we must understand what it means for “something to be before something else.” But this involves a proposition. Hence relational words mean only as constituents of propositions.⁶⁷

It is more correct, however, according to Russell, “to take an intensional view of relations, and to identify them rather with the class-concepts than with the classes.”⁶⁸ But this gives rise to a difficulty. If a relation taken in intension is identified with a class-concept, a relation, just as a class-concept, must have instances. Upon this matter, however, Russell states his position quite clearly. The “relation affirmed between A and B in the proposition ‘A differs from B’ is the general relation of difference, and is precisely and numerically the same as the relation affirmed between C and D in ‘C differs from D.’ And this doctrine must be held for the same reason to be true for all other relations; relations do not have instances but are strictly the same in all propositions in which they occur.”⁶⁹

Apparently this precision is only for the purpose of being contradicted, for we find him stating later⁷⁰ “we must suppose that we are acquainted with the meaning of ‘before’ and not merely with *instances* of it.” Unless he seems by “instances of a relation” the occurrence of a universal relation is a given complex, he has flatly contradicted himself. The very use of the word “universal” suggests instances, for a universal which does not have instances should not be so called. Furthermore it is clear that relations do have instances in the sense that there are kinds of relations. Accordingly time and space offer instances of a symmetric transitive relations.⁷¹

⁶⁷In an earlier article (M.T. 209) Russell maintained that it was possible to think of relations entirely apart from terms.

⁶⁸P. of M. 99.

⁶⁹P. of M. 51.

⁷⁰M.L. 213. The italics are mine.

⁷¹Russell’s argument (P. of M. 50) against the possibility of a relation having instances is as follows:

In the statements, $aD'b$, and

$eD''f$, the relations D' and D'' are assumed to be instances of the general relation of difference, D . Now for D' and D'' to be instances of D , they must have something in common. In order to have something in common, they must be related by the same relation to a given term. Hence, $D'Rg$, and

$D''Rg$. But this assumes that R in both cases is the same relation, which is contradictory to the original assumption. Hence D' and D'' can have nothing in common and cannot be considered as instances of D .

This proof contains a fallacy. There is a *petitio* involved in the assumption that for terms to have something in common they must have the *same* relation to a given term; they may have only *similar* relations.

The truth seems to be that, regardless of whether we do or do not assert that relations have instances, relations in intension are irreducible to any further notion. Russell speaks of them as "relations that occur in actual relational propositions."⁷² We may define them extensionally by classes and therefore by propositional functions, but the propositional functions themselves exhibit a relation between terms, so we commit a *petitio*. The result is, that just as "red" reduces to an ultimate "being red" which is called a predicate, so "before" resolves itself into an irreducible "something before something."

The final type of meaningful symbol need not detain us long; it is that type of symbol which is called by Russell "descriptive." Descriptive symbols are of two kinds, definite and indefinite. The former include all expressions of the form "the so-and-so" and all proper names in the non-logical sense of the term. "The names that we commonly use, like 'Socrates' are really abbreviations for descriptions:"⁷³ the name "Socrates" is a shorthand symbol for "the man who drank hemlock." The latter includes all expressions of the form "a so-and-so."

Descriptive symbols of both types are in the first place complex, which signifies that they are themselves composed of simple symbols, and that the meaning of the complex symbol is derivative from the meanings of the simple symbols, and in the second place incomplete, which implies, as we shall recall, two things: they have no meaning in isolation, and there is no constituent corresponding to the symbol as a whole. With regard to definite descriptions this has one important consequence. There is no constituent in the propositional-meaning corresponding to the descriptive word or phrase. As a result we are permitted to make assertions of the form, "God exists" or "The creator of the universe exists," even though there be no entity designated by these phrases. If such expressions were names, the propositions in which they occurred would lose all significance in case there were no such entity; we cannot name a non-existent. With regard to indefinite descriptions, the consequences are equally important. There is no constituent of the world which is symbolized by "a so-and-so." Consequently, "'I met a unicorn' . . . is a perfectly significant assertion, if we know what it would be to be a unicorn . . . Thus it is only what we may call the concept that enters into the proposition . . . there is not also, somewhere among the shades, something unreal which may be called 'a unicorn'."⁷⁴

The results of this chapter may be summarized as follows.
 (1) Proper names are simple and complete symbols. The "meaning" relation in this case is called "naming" and that

⁷²P. of M. 523.

⁷³L.A. 524.

⁷⁴L.M.P. 168.

which is named is always a particular. Proper names have meaning in isolation. (2) General names and adjectives are simple and incomplete symbols. They mean concepts, which refer either to universals or to classes. But membership in a class is determined only by a propositional-function. Hence the ultimate meaning is a predicative propositional-function. (3) Verbs are simple and incomplete symbols. They mean relational-concepts which refer to relations. But relation is defined only in terms of a propositional-function. Hence the ultimate meaning is a relational propositional-function. (4) Proper names in their non-logical sense, phrases of the form "the so-and-so" and "a so-and-so" are complex and incomplete symbols whose meaning relation is called "description." In neither case is there a single entity referred to, and in both cases the phrase has meaning only as an element of a propositional function.

CHAPTER THREE

THE PROPOSITION AS A UNIT

Early in our discussion of the previous chapter we found ourselves balked in the attempt to determine how words mean, by two difficulties. (1) Words mean in a number of different ways. (2) The way in which a given word means is dependent in general upon its function in the sentence. In any treatment of the meaning of propositions, two analogous difficulties arise. (1) Propositions mean in a number of different ways. (2) The way in which a given proposition means is dependent (in this case) upon the logical form of the proposition. The present chapter will attempt to overcome these difficulties by a classification of propositions into forms. We shall then be permitted in Chapter Four to turn to the direct problem of reference as applied to propositions.

We saw at the outset that every proposition-verbal is a complex. But every complex has a form, i.e. relation or relations between elements. Every proposition-verbal, therefore, asserts a relation between words. But it is apparent that we do not concern ourselves with the form of a proposition-verbal; such a form has no great importance logically. The form of all propositions-verbal is "aSbSc . . ." where "a," "b," "c," etc. are words, and "S" is the relation "immediate predecessor of."

It was shown, furthermore, that the first step in the analysis of any proposition-verbal is taken by grammar, when it asserts that every proposition permits of being resolved into subject and predicate. This, however, involves passing beyond the proposition as a form of words, and considers it from the point of view of syntax. Determination of syntax is consequently the first step in the determination of meaning. Verbally, an "S-P" proposition would be sharply distinguished from a "P-S" proposition; grammatically, the two are the same, since it is an accident of our language that "P" usually follows "S."

Syntax reveals an important consideration with regard to propositions. The meaning of a proposition cannot be determined in general by simply noting the *spatial* positions of its elements in the verbal formulation. There is a form possessed by the proposition, which is derived from the *logical* positions of its parts. The elements have logical relations one to another, which are derivable from but not identical with their spatial relations. An essential problem in the determination of meaning, is the ascertaining of the nature of these logical relations. They make up what is called the structure or *form* of the proposition. On the other hand, the elements which are so related are usually spoken of as the *content* or subject-matter of the proposition.

Form and content are defined as follows: Form is "the

way in which the constituents of the proposition . . . are put together. If I say, 'Socrates is mortal,' 'Jones is angry,' 'The sun is hot,' there is something in common in these three cases, something indicated by the word 'is.' What is common is the *form* of the proposition, not an actual constituent. If I say a number of things about Socrates—that he was an Athenian, that he married Xanthippe, that he drank the hemlock—there is a common constituent, namely Socrates, in all the propositions I enunciate, but they have diverse forms. If, on the other hand, I take anyone of these propositions and replace its constituents, one at a time, by other constituents, the form remains constant, but no constituent remains. Take (say) the series of propositions, 'Socrates drank the hemlock,' 'Coleridge drank the hemlock,' 'Coleridge drank opium,' 'Coleridge ate opium.' The form remains unchanged throughout the series, but all the constituents are altered. Thus form is not another constituent, but is the way the constituents are put together. It is forms, in this sense, that are the proper object of philosophical logic."⁷⁵

The distinction of matter and form as applied to propositions, however, may give rise to difficulty. For example, is the relation which enters into the proposition part of the content or part of the form? It seems to be the former because any analysis of a proposition reveals an actual element which is called the relating element and is symbolized usually in the proposition-verbal by a verb. It seems to be the latter because the essence of the relation is contained in the order of the terms related, and the order of the terms cannot be added on to the content of the proposition as another term. This might prove a rather serious difficulty were it not for the fact that in a rather obscure place⁷⁶ Russell makes a sharp distinction between a complex and a mere aggregate. A proposition is a complex and not an aggregate. A "complex differs from a mere aggregate of its constituents, since it is one, not many, and the relation which is one of its constituents enters into it as an actually relating relation, and not merely as one member of an aggregate." A proposition is therefore a unity which is complex. Yet this is not a paradox. The complexity lies in the fact that there are constituents: the terms and the relation; the unity consists in the fact that the relation gives a form to the whole which is thus constituted. Consequently the relation is part of the content of the proposition, for any enumeration of the elements of the proposition will contain the relation. But the relation is also a part of the form of the proposition, for any enumeration of the elements of the proposition even though such enumeration contains the relation itself, does not exhaust the

⁷⁵S.M. 43.

⁷⁶An article in *Mind*, 1910, "Some Explanations in Reply to Mr. Bradley." See also (M.T. 210; 348).

significance of the proposition; we must add the fact that the relation has sense or direction. Sense of a relation can never be given by simply naming the terms which are related, and then adding on the relation. The point to be emphasized here, for the sake of the metaphysical discussion which is to follow, is that although the relation is always an element of the proposition, it is not an element in the same sense that the terms related are.

It is the claim of logic, that form may be divorced from content and considered entirely by itself. We can discuss pure forms. Logical symbols represent the form of a propositional-meaning. Yet it must be remembered that this is not entirely true. Suppose, for example, it is claimed that " aRb " represent or describe the form of such a proposition as "Chicago is west of New York," where " a " symbolizes Chicago; " R ," west of; and " b ," New York. The reply to this contention must be that by such means we represent the form so far as it can be represented at all. But such a method fails as a complete explanation, simply because of the fact that the symbols themselves must have a form. In certain types of propositions the propositions cannot be symbolized without arranging the symbols in a certain order. And it is precisely this order of the elements, which makes up the form. Thus in the proposition cited, "Chicago is west of New York," we can represent the relation " aRb " but not " bRa " unless we change the symbol " R " to its converse. Hence " aRb " does not describe the form of the proposition; it simply symbolizes that for "Chicago," "New York," and "west" other entities may be substituted. The form remains undescribed. This is seen clearly in the fact that we cannot represent such a proposition by $(a,b)R$, or by " a and b and R ." There is something which the elements have in conjunction, which is lost the moment the elements are either separated or rearranged. The form is something which is given the moment the proposition is asserted. Wittgenstein⁷⁷ expressed this by saying that the proposition "shows forth" the form. But the form cannot be described or pictured or even talked about. It is indescribable and unutterable for every attempt at description of it must itself be made in a proposition, and this proposition will again have a form which demands description. The problem appears anew, therefore, of describing this form. Hence logical symbols cannot be considered as an ultimate representation of any proposition.⁷⁸

⁷⁷Wittgenstein, prop. 2. 172.

⁷⁸Wittgenstein has carried these ideas to their ultimate conclusion. Since all logic is pure form, and pure form cannot be spoken of at all, every proposition of logic is pure nonsense. Since philosophy and logic are one, philosophy ceases to be a "discipline" and becomes a method—a method of clarification of ideas.

Inasmuch as a proposition simply "shows forth" its form, the determination of what actual forms there are, must be a purely empirical matter. It is merely a process of collecting actual propositions which exist, and arranging them according to types. It can never justly claim to be exhaustive, because it can never be sure that it has *all* propositions. It can simply say: Here are the propositions which I have observed; they represent such and such general types of forms. If it asserts, for example, as traditional logic does, that all propositions can be put into the S-P form, and that this is an ultimate form, we must remember that such an assertion is merely a generalization from experience and liable to all the dangers of such generalizations. This classification and arrangement of forms is the empirical part of logic. Our knowledge of the forms of propositions is derived from experience.

Such a classification reveals certain general types of propositions:⁷⁹ atomic propositions, which are assertions about particulars; molecular propositions which are assertions of relations between atomic propositions; general and particular propositions; propositions of logic, which permit us to infer one proposition from another. This classification, however, does not claim to be exhaustive. There are quite likely other types of propositions which would fall into none of these classes. Russell suggests "A believes p" as being an example of such a type. Each of the general types enumerated has different forms: atomic propositions may express the relation of a quality to a thing, the relation of an individual to a class, the relations between two, three, or any number of things, etc.; molecular propositions include the hypothetical and disjunctive propositions of traditional logic, with an added conjunctive proposition, as well as modifications and negations of these; general propositions are classified together with particular propositions which are their negations; the general principles or propositions of logic include the principle of the syllogism, the principle of transitivity of relations, *reductio ad absurdum*, and all laws by means of which one proposition may be inferred from another. We shall limit ourselves in what follows to the first three types mentioned: the atomic, the molecular, and the general-particular propositions. A discussion of propositions of logic would lead us into the problem of the nature and meaning of inference, which demands a more strictly logical treatment than this discussion claims to be. Propositions involving "belief," "supposition," "hoping," etc., bring in extraneous psychological elements. Hence these two types will be omitted.

⁷⁹Boole (*Laws of Thought*, chap. iv) makes a distinction between two types of proposition called respectively, primary and secondary. This corresponds to Russell's distinction between atomic and molecular propositions. Boole merely considers the distinction logically, and does not carry out its necessary metaphysical implications.

The simplest type of proposition is that which is illustrated by a judgment of perception. It is called the "atomic" proposition because "there are other propositions in which atomic propositions enter in a way analogous to the way in which atoms enter into molecules."⁸⁰ Such propositions are called molecular. Atomic and molecular propositions are classed together as "elementary propositions."⁸¹ An atomic proposition is one which "asserts that a certain thing has a certain quality, or that certain things have a certain relation."⁸² A molecular proposition is one which is made up of atomic propositions connected by such notions as "if-then," "either-or," etc. An elementary proposition may therefore be defined in two ways. On the one hand it is a proposition which is either atomic or molecular; on the other it is a proposition "which does not involve such words as 'all,' 'some,' 'the,' or equivalents for such words."⁸³

It is hard even according to Russell himself to find a true example of an atomic proposition.⁸⁴ "A proposition such as 'this is red,' where 'this' is something given in sensation will be elementary,"⁸⁵ and, according to his later term, atomic. Elsewhere he gives such propositions as "this is before that," and cites Royce's example, "A gives B to C." On the basis of these, we might construct atomic propositions of any degree of complexity of relation. We thus have a whole hierarchy of propositions—those in which there is expressed a thing and a quality, two things and a relation, three things and a relation, four things and a relation, and so on. The totality of such propositions constitutes what Russell calls atomic propositions, and they are the simplest sort which experience reveals.

It must now be asked in what sense these differences of form among atomic propositions are ultimate. Russell's answer is that "you cannot reduce them downward, but you can reduce them upward."⁸⁶ That is, dyadic relations may be replaced by equivalent triadic relations, and triadic by tetradic and so on. Since qualities may be considered by analogy as monadic relations,⁸⁷ it is always possible to reduce qualities to dyadic relations. "This is white," for example, may be reduced to "this is similar to a," where "a" is something of standard quality of whiteness.⁸⁸ Thus we have done away with colors as qualities, and replaced them by relation "exactly like in

⁸⁰S.M. 52.

⁸¹Russell uses the term "elementary proposition" in his *Principia Mathematica*. He discards this expression in his later works for the two terms "atomic" and "molecular."

⁸²S.M. 52.

⁸³P.M. 95.

⁸⁴*Ibid.* 47.

⁸⁵*Ibid.* 95.

⁸⁶L.A. 527.

⁸⁷See above p. 22.

⁸⁸L.A. 35.

color.” This might be done with all predicates. But Russell asserts,⁸⁹ quite justly I think, that there is no reason to suppose that the actual relations of the world are of the order n rather than the order 1. Consequently reduction of relations of one order to those of a higher order, must be considered as complication rather than simplification.

With regard to the other matter, i.e. that you cannot reduce relations downward, there are two points to be mentioned. In the first place, the statement is not universally true—some relations *may* be reduced downward. In the second place, the belief that all relations can be reduced downward has led to the contention that every proposition is of the S-P form or at least can be reduced to that form. This has some marked metaphysical results. Let us consider first relations which may be reduced downward. This involves three cases, (1) symmetric transitive relations,⁹⁰ (2) symmetric intransitive relations, (3) relative products.

(1). Take the proposition “B is equal to C.” This may be reduced with some plausibility to two propositions of monadic order. One of these propositions will attribute a given magnitude to B, the other will attribute the same magnitude to C. Hence the two propositions “B is D” and “C is D,” where “D” is in both cases the same magnitude, are equivalents of “B is equal to C.” (2). Consider the proposition “B is different from C.” According to a similar pattern we can express this as “B is D” and “C is E,” where “D” and “E” are different attributes. (3). Although relations in general may not be reduced downward, it seems to me to be theoretically if not practically possible to reduce all relations of higher order than the order 2, to dyadic relations. Russell considers this only with regard to serial relations, i.e. the reduction of “B is before C which is before D” to the proposition, “B is before D.” Here the relations of B to C and of C to D are the same, and the relative product⁹¹ reduces to the square of the relation, which in turn is the relation itself. But this a special case of relative product; in its general form also it may assist us in the reduction of relations. Thus the proposition “B is the husband of C who is the daughter of D” may be reduced to “B is the son-in-law of D;” the former is a triadic relation, the latter is dyadic. It is true that there are actually very few cases where language offers a single word which symbolizes the produced relation, but that is an accident of language; the relative product may be formed in any case. For example language permits us to symbolize the relative product of the

⁸⁹*Ibid.* 36.

⁹⁰The clearest and least technical definition of these terms is to be found in S.M. 48-9.

⁹¹“The relative product of two relations R and S is the relation which holds between x and z when there is an intermediate term y such that x has the relation R to y and y has the relation S to z.” (P.M. 35.)

relation expressed in "B is eldest son of C who sits upon the throne D," as "B is heir to D;" but it does not permit us to simplify the relation of B to D where B is the brother of C who owns a dog D. A dog may be as valuable a possession as a throne, but it has not been so important historically. Consequently language does not help us out. But the fact remains that even in this case, B does have some relation to D. I see no reason why, upon this basis, all relations of higher order than 2, might not be reduced to dyadic relations. That dyadic relations may not always be reduced to monadic relations, is due to a significant fact which we have now to consider.

Since it is upon this point that Russell following Peano and Frege breaks sharply with traditional logic, it will be well to consider it at greater length. The old logic "believed that there was only one form of simple proposition," (i.e. of propositions not stating a relation between two or more other propositions) namely, the form which ascribes a predicate to a subject. This is the appropriate form in assigning the qualities of a given thing—we may say 'this thing is round, and red, and so on.' Grammar favours this form, but philosophically it is so far from universal that it is not even very common. If we say 'this is bigger than that,' we are not assigning a mere quality of 'this,' but a relation of 'this' and 'that.' We might express the same fact by saying 'that thing is smaller than this,' where grammatically the subject is changed. Thus the proposition stating that two things have a certain relation, has a different form from subject-predicate propositions, and the failure to perceive this difference, or to allow for it, has been the source of many errors in traditional metaphysics."⁹² The reference, of course, is to the problem of the existence of relations. Since relations are assumed to be reducible to attributes, either of the terms related or of the whole made up of the terms, there is no longer any need of assuming that relations exist; they can be reduced to the "nature" of that which is related.

There have been three attempts to carry out this reduction and Russell tries to show that all of them have been unsuccessful.⁹³ Let us take for our example, "B is greater than C." The first view attempts to do away with the relation by making "is greater than C" an attribute of B. The propositions would be represented as "B is (greater than C)." But this fails as an adequate explanation, Russell shows, for the adjective which is predicated of B, proves to be complex; it consists of the parts "greater" and "C," both of which are essential. To say that the predicate of B is merely "greater" destroys the meaning; the proposition asserts "greater than C." Thus the adjective "greater" has its full meaning only in its reference; but this

⁹²S.M. 45.

⁹³P. of M. 222-226.

is simply a round-about way of saying that "greater" is relative to "C." Hence we have not reduced the relation.

Two other arguments might be adduced in support of Russell's position. The first follows from the consideration just given. If "greater than C" is to be predicated of B, in the same sense that "red" or any other color is, it is clear that though the relation of predication is the same in both cases, the nature of the predicate in each case is different. Consequently we should be forced to divide predicates into two types: relational and non-relational, and the distinguishing mark between them would be presence of a relation in one case and absence in the other. But this differs from Russell's position only in the matter of terminology. He calls "relational predicates" simply "relations," and "non-relational predicates" by the term "predicates" or "attributes." The reduction, therefore, is not accomplished. The second argument arises out of the difficulty of handling inference of the type "A is greater than B; B is greater than C; therefore A is greater than C." This type of inference traditional logic has fittingly called extra-syllogistic, because it fails to fit into the syllogistic or S-P form of inference. If "A is greater than B" can be expressed as "A is (greater than B)," it seems reasonable to suppose that the inference given above could be reduced to the syllogism. All attempts to do so, however, seem to have been in vain. Hence such a proposition is not of the S-P form.

The second attempt at reduction of relations to attributes, is that which supposes the relational proposition to be resolved into two monadistic propositions, each attributing a quality to one of the related terms. This is the procedure which was adopted in reducing symmetric relations to attributes, and we saw that in such a case the argument could be given some plausibility. But the case is different with asymmetric relations. Let us assume that the given example is resolved by attributing a given magnitude to B, and a lesser magnitude to C. But this does not exhaust the meaning of the original; we need to assert also a relation between the magnitudes, and this would again be an asymmetric relation which demands resolution. We cannot assume mere difference of magnitude, for this fails to give account of the sense of the relation. Hence this attempt which was adequate for symmetric relations fails when the relations are asymmetric.

The third attempt (the monistic) tries to show that the two terms "B" and "C" may be combined into a whole of which we can predicate the relation "greater." But it is apparent that this again loses the sense of the relation. If B and C simply compose the whole, then the order or position of the terms is insignificant, i.e. we could assert either "(BC)R" or "(CB)R". But in asymmetric relations, it is just this posi-

tion which is essential; we assert "BRC" and not "CRB." Hence any attempt to resolve relations by making them attributes of a whole composed of the elements related, fails unless the terms have position in the whole. But position is again relation, and we have not resolved the relation.

It is easy to see how such a view has led Russell to an atomism of propositions.⁹⁴ Just as physical atoms are, or were at one time supposed to be, ultimates of physical analysis, so propositions are ultimates of logical analysis. Just as an atom of H differs from an atom of O, and cannot either be derived from it or reduced to it, so a subject-attribute proposition in general differs fundamentally from a dyadic relational proposition, and cannot by any sort of trick be derived from it. Russell is not certain as to whether one atomic proposition may be inferred from another;⁹⁵ he is quite certain, however, that there are atomic propositions of such a nature that an equivalent proposition of a different type can never be found. They are ultimates, therefore, in this sense.

A further examination of atomic propositions reveals a distinction among them on the basis of which we may classify all such propositions into two groups: the one is called positive atomic propositions; the other is called negative atomic propositions. This distinction is properly discussed in connection with the problem of truth and falsehood and will be treated again in that connection. But certain facts about positive and negative propositions may be noted here.

(1). On the verbal side, it is always easy to make the distinction: atomic propositions-verbal which contain the word "not" are negative; those in which such a word is lacking are positive. As long as we limit ourselves to atomic propositions this test is sufficient. But as we shall see, it fails with universal-particulars propositions which are contradictories of each other, but either of which may be considered positive.

(2). We must not expect to reduce the idea of negation to any more fundamental notion, for negation is one of the indefinables of logic.⁹⁶ The negative proposition does not depend "on a cognitive subject for its definition."⁹⁷ It cannot be reduced, that is, to a disbelief in the corresponding affirmative proposition, for belief and disbelief are psychological matters and themselves derived from the notion of truth and false-

⁹⁴Russell is not consistent with his use of the term "atomism." Sometimes he uses it to refer to propositions as ultimates, and sometimes to particulars as constituents of propositions. Perhaps in order to keep the parallelism complete, one ought to speak of propositions as atomic and particulars as "electronic." Such use would have the advantage of avoiding ambiguity.

⁹⁵S.M. 53. If we are to accept Wittgenstein's statement, no atomic proposition can ever be inferred from another.

⁹⁶P.M. 97.

⁹⁷L.A. 42.

hood.⁹⁸ The distinction positive-negative, however, is equivalent to the disjunction affirmation-denial,⁹⁹ and both are equivalent to the opposition of truth and falsehood. Thus "p" is equivalent to the affirmation of p, and this is equal to "p is true." On the other hand, "not-p" is equivalent to the denial of p, and this equivalent to "p is false." Or since "not-p" is the same type of proposition as "p," we may also say on the one hand that "not-p" is equivalent to the affirmation of not-p, and this is equivalent to "not-p is true;" on the other hand "not-not-p" or "p" is equivalent to denying not-p, which is equivalent to "not-p is false."¹⁰⁰ What this equivalence amounts to, then, apart from any notion of affirmation and denial, is just this: if a positive proposition is true, the corresponding negative will be false; if a negative proposition is true, the corresponding positive proposition is false. This is evidently equivalent to the law of contradiction: The joint assertion of p and not-p is false.

(3). A negative proposition has no different status from that of a positive proposition; the word "not" conveys no distinction upon the proposition in which it occurs. A negative proposition is just as "simple" as a positive proposition and may therefore be "taken at its face value."¹⁰¹ A negative proposition "not-p" cannot be reduced to some proposition "q" which is true and incompatible with "p." Negative propositions are ultimate in the same sense in which positive propositions are. "Propositions . . . are always themselves positive facts" though differing among themselves "by the absence or presence of the word 'not'."¹⁰²

(4). The negative word cannot be taken as a qualification of the predicate, i.e. one cannot say that the notion, "not red," is equivalent to the notion, "not-red." The reasons given for this are two in number.¹⁰³ (1) It fails in non-attributive propositions. This needs no comment. (2) It fails when a quality is denied of a subject which does not exist. For example if we take the proposition "The present King of France is bald"—a false proposition—and contradict it according to the view which we are examining, we shall have "The present King of France is not-bald." But if the original proposition is false, this must be true and the present King of France must possess hair. Since this is not the case we must consider the "not" as modifying the proposition as a whole, and not simply the predicate term. It is properly indicated as "not: the present King of France is bald."

⁹⁸H.P.M. 43.

⁹⁹Russell finds it impossible to decide whether "not-p" is anything other than "p is false." (M.T. 523.)

¹⁰⁰Assuming DeMorgan's theorem.

¹⁰¹L.A. 43.

¹⁰²H.P.M. 39.

¹⁰³L.A. 43-4.

(5). Negation cannot be resolved into a supposedly simpler notion such as "incompatibility." It is sometimes said, for example, that the proposition "this is not red" asserts a relation of incompatibility which exists between "this" and "red." But such a procedure does not do away with the notion of negation, for all attempts to define or explain what we mean by incompatibility involve the negative. If we say that the negation in the proposition "the table is not square"¹⁰⁴ is due to the incompatibility existing between roundness and squareness, this incompatibility can itself be explained only by such a proposition as "round is not square."

The second type of elementary propositions, what Russell calls the molecular proposition, is of a distinctly different form. "'Molecular' propositions are such as contain conjunctions—if, or, and, unless, etc.—and such words are the marks of a molecular propositions."¹⁰⁵ They are called molecular propositions because "they contain other propositions which you may call their atoms."¹⁰⁶ Examples are easy to find: "If this is white, that is blue" (hypothetical proposition). "Either B is before C or B is after C" (disjunctive proposition). "This is large and that is small" (conjunctive proposition). "Unless B is round, B is square" (hypothetical proposition with negative antecedent).

Certain characteristics are immediately noticeable with regard to this type of proposition. In the first place, molecular propositions contain more than one verb; atomic propositions always contain only one verb. Yet this must not be taken as a distinguishing mark of molecular propositions. There are other propositions containing more than one verb which are not molecular. Such a proposition, for example, as "John believes that they will come," contains the two verbs "believes" and "come." Yet it is not molecular. It is a type of proposition which is neither atomic nor molecular, and which therefore demands special treatment. True molecular propositions, on the other hand, contain atomic propositions which are, so to speak, on the same level and not contained one in the other as in the example given. Each atomic element of a molecular proposition has meaning by itself.

In the second place, and this is the main distinction between atomic and molecular propositions, they differ in the way in which their truth and falsehood is determined. Since this discussion belongs properly to our treatment of proposition as symbols, I shall only suggest the distinction here. Whereas the truth of an atomic proposition lies immediately in something outside the proposition itself, in a molecular proposition the truth depends directly upon the truth of the atomic propositions which constitute it, and therefore indirectly upon an

¹⁰⁴H.P.M. 4-5.

¹⁰⁵S.M. 54.

¹⁰⁶L.A. 37.

external fact. Molecular propositions are consequently called "truth-functions"¹⁰⁷ of atomic propositions, because the truth value (i.e. truth or falsehood) of the molecular proposition depends upon, or is a function of, the truth or falsehood of the atomic propositions. An atomic proposition is therefore the argument to a molecular proposition.

In the third place, whereas atomic propositions are independent of one another, molecular propositions may be reduced to one another. "If this is white, that is blue," is equivalent, for example, to "Either this is not white, or that is blue,"—that is, implication is reduced to conjunction. Again the same proposition may be reduced to "The supposition of this being white, is incompatible with the supposition of that not being blue." Here implication is reduced to incompatibility. There is no reason, so far as I can see, why any one or two of these ideas should be taken as fundamental, and the others as derivative. For Russell the ideas of conjunction and negation are considered to be underived, and the rest are deduced from these. Sheffer and Nicod have subsequently shown that one idea, that of incompatibility, will suffice in the place of Russell's two ideas. That, however, is of no great importance for our discussion. The essential point here is that molecular propositions are not independent as atomic propositions, but may be reduced one to another.

It is evident in the fourth place, that molecular propositions form the basis upon which all inference is carried on. By means of molecular propositions, we pass from the truth of one atomic proposition to the truth of another. Atomic propositions, we observed, are in all probability at least independent of one another, as far as inferring from the truth of one to the truth or falsehood of another is concerned. The only way to infer from one atomic proposition to another is to have both atomic propositions contained as elements in a molecular proposition. But it is not the atomic propositions alone which make up the molecular proposition; it is the atomic propositions together with the ideas signified by "if-then," "either-or" and the like. It is these notions which give the essential nerve of the inference.

The third type of proposition which we have to consider is that called by Aristotelian logic, "universal and particular propositions." These according to the traditional logic, class themselves into two groups: those asserting about "all" or "none" of a group, and those asserting about "some." The former are called universal or general, the latter are called particular. They are, furthermore, each other's negations. "Some men are not good" is the negative of "All men are good;" "Some men are good" is the contradictory of "No men are good."

¹⁰⁷L.A. 40.

Russell agrees to this general distinction of universal-particular propositions. But with regard to the distinction of positive-negative, he emphasizes the fact that it is purely a matter of taste which we regard as positive and which negative. The tendency, on the one hand, is to consider general propositions as affirmative, and particulars as negative. But this is evidently due to the fact that we commonly start with universal propositions and pass to their contradictions which are particular. On the other hand, since, as we shall see immediately, general propositions do not assert existence whereas particular propositions do, one would feel inclined to consider general propositions as negative and particulars as affirmative. Thus either may be considered as positive, depending upon the point of view. Hence it is best, Russell suggests,¹⁰⁸ to do away with the distinction entirely, and to speak only of general propositions on the one hand, and particular propositions or existence propositions on the other.¹⁰⁹ It is evident that the mark of negative propositions which we noted in our discussion of atomic propositions fails here; the word "not" cannot be taken as indicative of a negative proposition of this type.

The key to the interpretation of this type of proposition lies in the propositional-function. We saw¹¹⁰ that "sentences involving such words as 'all,' 'every,' 'a,' 'the,' 'some' require propositional-functions for their interpretation."¹¹¹ It can best be shown how this is done by reverting to this notion of propositional-function.

A propositional-function is a form of propositions containing at least one variable. It becomes significant only when the range of values of the variable is determined. A propositional-function for which the range of values is determined, is a proposition. There are three possibilities as to a propositional function: it may be true for all, some or no values of the variable. The third of these need not be considered as a separate case, for if we admit the validity of obversion, a propositional-function true for no values of the variable may be reduced to the corresponding negative propositional function which is true for all values. The supposedly exceptional case of "true for one value only," is included under "true for some values."

On the basis of this notion, the traditional A, E, I, and O propositions reduce to the following: "All men are mortal" becomes "'x is human implies x is mortal' is true for all values of x." "No men are mortal" becomes "'x is human implies x is immortal' is true for all values of x." "Some men are mortal" becomes "'x is human and mortal' is true for some

¹⁰⁸L.A. 191.

¹⁰⁹In one place (M.L. 55) Russell calls particular propositions negative, and universal propositions affirmative.

¹¹⁰See above p. 13.

¹¹¹I.M.P. 158.

values of x ." "Some men are not mortal" becomes "' x is human and immortal' is true for some values of x ."

Certain points may be noted with regard to this reduction. (1). The A and E propositions do not differ in form except by the substitution of a negative predicate function for an affirmative one. The same is true of I and O. (2). Since an implicative relation may be expressed also as disjunction and conjunction, the first of the propositions given above may be expressed as "'Either x is inhuman or x is mortal' is true for all values of x ," and "' x is human and immortal' is true for no values of x ." (3). Contradiction of a proposition of this type involves not only negating the propositional-function as such, but also contradicting the range of values of the variable. If Fx represents any function of x , the contradiction of " Fx :all" is "not- Fx :some;" the contradiction of " Fx :some" is "not- Fx :all." Thus "we can if we choose, take one of the pair 'always,' 'sometimes' as a primitive idea and define the other by means of the one and negation." Or we can "take both 'always' and 'sometimes' as primitive ideas, and define by their means the negatives of the propositions in which they occur."¹¹² This latter procedure Russell considers to be more nearly correct. (4). If this interpretation of general propositions is correct, it is no longer possible to identify individual propositions of the form "Socrates is a man" with the universal propositions of the form "All men are mortal" as traditional logic does. They are of essentially different types. The former, assuming for the moment that "Socrates" is a name in the logical sense, is an atomic proposition of the subject-attributive type, whereas the latter reduces to an implicative relation between propositional-functions asserted as true for all values of the variable. The essential distinction is that elementary propositions contain no apparent¹¹³ variables, whereas universal propositions must be interpreted upon the basis of propositional-functions, and propositional-functions always contain variables. Traditional logic is therefore wrong in assuming that these two types of propositions are essentially similar in structure.

This much is clear, and can hardly be questioned. But the further developments of this view lead to some difficulties. For on the basis of this interpretation of universal-particular propositions, consistent with his attempt to "derive a metaphysic from a logic," Russell passes to a definition of the concept "existence." Any attempt to define such a term tends at the outset to arouse suspicion; one feels with Aristotle that we can talk of its "aspects" but that *being* itself because of its universality must always escape definition. The suspicion is supported when we note that "existence" is defined by Russell

¹¹²I.M.P. 160.

¹¹³For the distinction between real and apparent variables see P. of M. 13.

in terms of a propositional-function which is true for some values of the variable—a notion which, on the surface at least, appears to be of greater complexity than the idea of existence itself. When we recall that a propositional-function is “nothing,”¹¹⁴ it seems strange that by adding the notion of “sometimes” to that which is nothing we can produce that which is in a sense “everything.” Yet our suspicion may prove to be unwarranted; Russell’s position deserves at least an impartial consideration.

His contention is as follows: “When you take any propositional-function and assert of it that it is . . . sometimes true, that gives you the fundamental meaning of ‘existence.’ You may express it by saying that there is at least one value of x for which that propositional-function is true. Existence is essentially a property of a propositional-function.”¹¹⁵ The fundamental form of existence “is that which is derived immediately from the notion of ‘sometimes true’ . . . This is the fundamental meaning of the word ‘existence.’ Other meanings are either derived from this or embody mere confusion of thought.”¹¹⁶ Such a dogmatic and pretentious claim does not invite criticism; it is possible, however, that we may find some slight “confusion of thought” even in Russell’s own position.

Let us take his example. When one says “There are men” or “Men exist” he means to assert that the propositional-function “ x is human” is true for at least one value of x . This is supposed to constitute a definition of the notion “Men exist.” Abstracting from this example, the general notion of “existence,” we may define the concept as “a propositional-function of one variable which is true for at least one value of the variable.” The notion of existence, therefore, is a compound of four ideas: (a) propositional-function, (b) variable, (c) truth, (d) value of a variable.

It is not necessary to make a detailed examination of these notions in order to show the circular nature of Russell’s definition. (a) Take the idea of propositional-function. In spite of the statement that propositional-functions like propositions are nothing,¹¹⁷ they are evidently something, i.e. they are facts. Facts, however, make up the world and are among the obvious elements of experience.¹¹⁸ Evidently, then, the mere mention of a propositional-function assumes some sort of existence, that is, the existence of facts. (b) Consider the notion of variable. A variable is defined as follows: “Taking *any* term, a certain member of any class of propositions of constant form will contain that term. Thus x , the variable, is what is denoted by

¹¹⁴L.A. 196.

¹¹⁵L.A. 195.

¹¹⁶I.M.P. 164.

¹¹⁷L.A. 62.

¹¹⁸*Ibid* 500.

any term.”¹¹⁹ The character *x*, then, is simply a symbol in the same sense that any word is a symbol, except that it denotes ambiguously. It symbolizes something, though we should have difficulty in saying what. But that which it symbolizes, whatever it may be, must have some sort of existence. Consequently we cannot employ the notion of a variable in a definition of existence, for every variable has its meaning in the fact that it symbolizes something which “exists” (c) The notion of truth exhibits the defect of Russell’s definition even more clearly. For the conception of truth and falsehood always involves reference to a fact—direct reference if a proposition is true, and indirect reference if a proposition is false. The notion of factual existence is therefore indissolubly bound up with the notion of truth. Hence to say that a propositional-function which is sometimes true *defines* existence is clearly a circular procedure. (d) What, finally, can we say about the value of a variable? The value or argument of a variable is that which satisfies the variable, i.e. makes the proposition in which the variable occurs a true proposition. But where do we get any possible argument of a variable? It is evidently not drawn from nowhere but exists, either at a time and place, or in a logical world which is neither temporal nor spatial, or in some other realm if only the world of “pure fancy.” We say that “*x* is a man” is true for certain values of *x*, namely those which are men. That is, if we put a “man” in place of “*x*”, we shall have a true proposition. But this is impossible unless we have a man; the concept of existence is therefore prior to the notion of a propositional-function being sometimes true.

It is possible however that Russell would deny that he had ever attempted to *define* the notion of existence. It is hard to conceive what interpretation we could give to the phrase “fundamental meaning of existence” other than that of “definition of existence.” Let us suppose, however, that even though a propositional-function does not *define* existence, it at least *describes* existence in its fundamental form. According to Russell, then, all other notions of existence may be derived from this. But such procedure will evidently involve a *petitio*. For we have seen that “the fundamental notion of existence” involves in its very meaning all possible notions of existence. Since a variable is that which denotes any term, and “term,” according to our previous discussion,¹²⁰ is the “widest word in the philosophical vocabulary,” the existence implied when we mention a variable can hardly be called “fundamental.” It is obviously impossible to derive “other meanings” of a given word from a “fundamental meaning” if the attempt to describe the latter implicitly involves the former.

One thing is at least certain. If it is not true that Russell

¹¹⁹P. of M. 89.

¹²⁰Chapter ii.

attempts to define existence, and if it is questionable whether he would attempt to derive other meanings of existence from a fundamental notion which is concerned with propositional-functions, there is little doubt but that there is still one sense in which the notion of a propositional-function as sometimes true does involve existence. "General propositions are to be interpreted as not involving existence. When I say, for instance, 'All Greeks are men,' I do not want you to suppose that that implies that there are Greeks. It is to be considered emphatically as not implying that. That would have to be added as a separate proposition."¹²¹ On the other hand "the contradiction of any general statement does assert existence." Particular propositions assert existence; general propositions do not. This is the basis for Russell's designation of particular propositions as existence propositions.

The most important consequences of this notion lie in the fact that if it is true much of traditional logic must be relegated to the scrap-heap. All inference from general propositions to particular propositions become unwarranted, for the latter contain an existential factor not found in the former. On the other hand much that is obscure in the Aristotelian logic is made intelligible, particularly the difficulties in connection with distribution of terms. This leads one to suspect that Russell's contention is in some sense correct. I have shown elsewhere¹²² what I consider this interpretation to be. In general it involves a sharp distinction between the intensional and the extensional points of view in explaining the significance of general and particular propositions. From the extensional side, i.e. from the aspect of derivation, both general and particular propositions contain an existential element; from the intensional or meaning side, neither type contains such an element. Russell's mistake lies in giving the intensional interpretation to general propositions, and the extensional interpretation to particular propositions. A true logic will be purely intensional and will have no regard for existence. Existence, we saw above, can never be defined or reduced to any simpler notion; it is therefore not intensional. It is an empirically given notion just like Wittgenstein's concept of object.¹²³

¹²¹L.A. 191.

¹²²Cf. my article, "To Exist or Not To Exist," *Monist*, Apr. 1926.

¹²³*Tractatus Logico-Philosophicus*. Prop: 4.127.

CHAPTER FOUR

THE PROPOSITIONAL AS A SYMBOL

Having put aside the question of word and propositional meaning, we are now ready to turn to the problem of objective reference. We shall have to consider this first from the point of view of words, as the proposition is a complex of which the words are elements. With regard to proper names there is no difficulty for the objective reference is the meaning. As concerns general names and verbs, we found that the ultimate meaning lies in a propositional-function, either of the predicative or the relational type. Since the situation is analagous in the two cases, we may confine ourselves for the purpose of illustration to the general name.

What is there actually involved in the structure of the world, which corresponds to such a word as "red"? The ultimate meaning of such a word is a propositional-function of the form "x is red." But a propositional-function becomes a proposition only when the range of values is determined. Such a propositional-function may be true for all, some or no values of x. The corresponding propositions would be respectively: "Everything is red," "Something is red" and "Nothing is red." Now the truth of a proposition, as we shall see, is determined by the existence of a fact of which it is a picture. Hence the first of these propositions will be true if it is a fact that all things are red; the second will be true if there is at least one thing which is red; the third will be true if there is not a single red thing in existence. It is of no consequence at this point which of these forms we give to the propositional-function. The important consideration here is that *the existential status of a "predicate" is indissolubly bound up with the existence of a fact.*¹²⁴ We become aware of a predicate only when it exists as an element in a fact. We can never know of the existence of "red" without knowing something which "is red." This means, simply, that if we take preceptual experience as the final criterion of existence, it is on purely inferential grounds that we assume an independent existence of universals; their existence as elements of a complex does not warrant our concluding immediately that they exist isolated. The same must be true of relations. We are acquainted with relations only in conjunction with terms which are related; we know a relation only as a constituent of a complex. Consequently its existence as isolated must be inferential, and we can never be aware of it in the same way we are aware of a particular.

It becomes clear, then, what Russel has in mind when he asserts of the structure of the world that universals "are an

¹²⁴It must be remembered that Russell uses the word "predicate" to denote both the meaning and the objective reference. See note p. 19.

inferred part, not a part of our data."¹²⁵ (It is evidently permissible to retain Russell's use of the word "universal" if we remember that universals include predicates and relations.) But how, in view of the fact that universals exist only by inference, can we be "immediately aware" of them,¹²⁶ since acquaintance is a relation which is "without the intermediary of any process of inference"?¹²⁷ This is one of those direct contradictions in Russell's philosophy which indicate a deliberate change of point of view. The only possible solution, it seems to me, lies in assuming that by awareness of universals Russell means awareness of a fact of which a universal is, by analysis, an element. This interpretation is at least consistent with his most recent view that "a universal never appears before the mind as a single object in the sort of way in which something perceived appears."¹²⁸ A particular may exist as isolated, and consequently may be perceived apart from relations to other particulars. But, if our analysis has been correct, we can never affirm dogmatically that a universal exists in isolation; it always *appears* in conjunction with a particular. We become aware of universals by observing a large number of facts containing universals, i.e. by observing a sufficient number of particulars which may be said to "be universals" in the predicative sense. Thus we are said to conceive universals.

Now let us consider the metaphysical results of our analysis. It becomes possible to make a two-fold division of reality on the basis of the distinction between particulars and universals. Particulars are those elements of the world "which enter into complexes only as subjects of predicates or the terms of relations, and, if they belong to the world of which we have experience, exist in time, and cannot occupy more than one place at one time in the space to which they belong." Universals are those elements "which can occur as predicates or relations in complexes, do not exist in time, and have no relation to one place which they may not simultaneously have to another."¹²⁹ "Thoughts and feelings, minds and physical objects . . . exist . . . But universals do not exist in this sense; we shall say that they *subsist* or *have being*, where 'being' is opposed to 'existence' as being timeless. . . The world of being is unchangeable, rigid, exact, delightful to the mathematician, the logician, the builder

¹²⁵In the example "yellow differs from blue" (M.L. 212) we have apparently a case of universals which are terms of a relation, and therefore occupy the position of particulars. In view of Russell's later reduction of non-relational universals to predicates, this proposition would have to be expressed in somewhat the following form: "'x is yellow and y is blue' implies that x differs from y" is true for all values of x and y. The quotation given in the text (A.M. 228) makes this reduction necessary.

¹²⁶M.L. 212.

¹²⁷P. of P. 73.

¹²⁸A.M. 228.

¹²⁹Aristotelian Soc. Pub. '11-12. "On the Relation of Universals and Particulars."

of metaphysical systems, and all who love perfection more than life. The world of existence is fleeting, vague, without sharp boundaries, without any clear plan or arrangement."¹³⁰ Russell has had little to say of this world of subsistence since the publication of his *Problems of Philosophy*. Consequently, in view of his temperament, it would be a rash assumption to assert that he would still insist upon this ultra-physical world. One passage in his more recent works¹³¹ suggests that he continues to favor some such division of reality. He says that the object of which mathematics speaks is "a logical object, not a particular, not a thing like a lion or a unicorn, but an object like a function or a number, something which plainly does not have the property of being in time at all." It must be emphasized that a universal does not exist in the sense that a unicorn is said to exist. Unicorns, for Russell, do not exist. "Logic must no more admit a unicorn than zoology can."¹³² The important point is that the type of existence exhibited by universals is essentially different from that exhibited by particulars. "When I say of a universal that it exists, I should be meaning it in a different sense from that in which one says that particulars exist. . . . The statement concerning any particular that it is not a universal is quite strictly nonsense—not false, but strictly and exactly nonsense. You never can place a particular in the sort of place where universal ought to be, or vice versa."¹³³

It is necessary, however, to make a distinction within the realm of universals. Not only do particulars and universals have different "types" of existence, but universals differ among themselves. The most apparent distinction is that between predicates and relations. We saw¹³⁴ that certain types of dyadic relations, those which are asymmetric, could never be reduced to predicates; this suggests that predicates and relations are of fundamentally different types. On the basis of this "typical" distinction, Russell finds it necessary to assume that predicates and relations have correspondingly different types of existence. Just as you can never replace a particular by a universal, so you can never replace a predicate by a relation. Their nature as functional elements of propositional-meaning determines their metaphysical status.

This brings up the question of the doctrine of types, to which we must now turn. On the basis of this theory, the world is further complicated so as to include types of predicates and types of relations.

The doctrine of types¹³⁵ is a logical theory. It is a hypothetical construction devised to facilitate the solution of certain

¹³⁰P. of P. 156.

¹³¹L.A. 348.

¹³²I.M.P. 169.

¹³³L.A. 350.

¹³⁴See above pp. 32-34.

¹³⁵P. of M. appendix B; P.M. 63-68; L.A. 351-362.

logical problems which take the form of contradictions. If we admit the theory of types, the contradictions disappear. This is its only logical justification. The one other reason for accepting it, is that the "theory has a certain consonance with common sense which makes it inherently credible."¹³⁶ Three considerations will serve to bring the notion before us.

(1) Is or is not a class a member of itself?¹³⁷ There are evidently three possibilities: (a) All classes are members of themselves. (b) Some classes are members of themselves, and some are not. (c) No class is a member of itself. (a) If all classes are members of themselves, then every class will have at least one member. This results in a contradiction, for the null class must have at the same time no members and one member. (b) If some classes are members of themselves, then we may put all classes into one of two groups: those which are members of themselves (call this A) and those which are not members of themselves (call this B). Then it becomes a problem what to do with B; is it or is it not a member of itself? If it is, then it belongs within B and becomes thereby a class which is not a member of itself; if it is not, then it belongs with A and becomes thereby a class which is a member of itself. In view of the contradiction this possibility must be rejected. We are left with assumption (c) that no class is a member of itself.

(2). It is possible to prove that there are more things in the world than there are things in the world. Suppose the number of things in the world to be n ; then the number of classes which can be formed out of these things is 2^n . But 2^n is always greater than n . Consequently the number of things (classes) which exist, is greater than the number of things which exist.

(3). Take the proposition, "All propositions are false." If this is true, the proposition itself must be false, and hence some propositions are true. Therefore the propositions "All propositions are false" and "Some propositions are true" are both true.

There are, of course, a large number of other forms which these contradictions take.¹³⁸ One of these concerns relations: May a relation R have the relation R to itself? Another is about Epimenides and the Cretans. But the solution in every case is afforded by a resort to the same principle, the theory of types.

¹³⁶P.M. 39.

¹³⁷Elsewhere (P. of M. 102) Russell states the problem in the following form: May or may not a predicate be predicated of itself? This is the same problem from the intensional side.

¹³⁸See "*The Paradoxes of Mr. Russell*," E. R. Guthrie, 1915, also, "Russell's Theory of Types," Guthrie, in Jour. Phil., Psy. Sci. Meth. 1915.

The first two contradictions given above lead to the following logical construction, which is one aspect of the doctrine of types. Classes and particulars must be distinguished fundamentally; a class of particulars is not in turn another particular; the two are on different levels of existence. It is impossible that is, to add together classes and individuals and get as a result a new class. One can never put a particular where a class belongs or vice versa. This simply reinforces the results of our consideration regarding particulars and predicates. But there is a further development. For classes of classes are on a different level from classes of particulars. This leads to a hierarchy among predicates. If "red" is the class of things, and "blue" is the class of blue things, and so on, then "color" is the class made up of red, blue and all other colors; this makes "color" a class of classes. Hence "red" and "color" have different types of existence.

Analogous arguments hold for relations. Particulars and relations can never be exchanged in any proposition; nor can predicates and relations. Relations which relate classes are on a different level from relations which relate particulars, and each of these has a different status from that of relations which relate relations. There is thus a hierarchy of relations, with each level possessing a distinct type of existence.

The third contradiction leads to an extension of the doctrine of types to include propositions, and derivatively facts. Given any propositional-function, the values which make the propositional-function significant must be drawn from a different level from that occupied by the propositional-function itself. Hence we have an order of functions: functions about particulars, functions about functions about particulars, and so on, called first order functions, second order functions, etc. When we have a proposition, therefore, such as "All propositions are false" this proposition cannot itself be included among the totality of "all propositions" for this proposition is on a different level.

The ultimate result of our analysis is a world of extremely great complexity. One is usually prepared for some degree of complexity at the end of a process of analysis. Russell's world is appalling in its intricacy. The "ultimate simples" out of which the world is built are not only infinite in number, but "are of an infinite number of sorts. There are particulars, qualities, and relations of various orders, a whole hierarchy of different sorts of simples, but all of them, if we were right, have in their various ways some kind of reality that does not belong to anything else."¹³⁹ But the complexity exists not only in the number and kinds of particulars. We shall see immediately that there is also an infinite number of kinds complexes, i.e. facts. Fortunately, except for these facts, everything above the level

¹³⁹L.A. 365.

of particulars has an "inferred" existence, so that unless we are impressed with the strength of the grounds for the inference, we need not burden our world with this "metaphysical lumber." But since predicates and relations do actually exist as elements in facts, and since facts are the most obvious elements of the world¹⁴⁰ we must turn to a consideration of this aspect of reality.

The problem of the relation between propositional-meaning and its objective reference is commonly called the problem of truth and falsehood, and it is important to remember that it is a purely theoretical problem. Russell is not concerned with asking how we can know whether a proposition is true or false; he is not attempting to answer the problem, "What is the test of truth?" He is turning his efforts rather to the abstract problem of determining what is meant by the question as to whether a proposition is true or false; he is trying to answer the question, "What is truth as such?"¹⁴¹ The failure to distinguish between these problems leads, as Russell suggests, to an answer which is not applicable to either. By confining his attention to the theoretical problem, he is avoiding a possible attack; he cannot be criticised upon pragmatic grounds of constructing a theory which does not work. He says, for example, that "the formal definition of truth by correspondence of a proposition with its objective seems to be the only one which is theoretically adequate. The further inquiry whether, if our definition of truth is correct there is anything that can be known, is one that I cannot now undertake; but if the result of such an inquiry should prove adverse, I should not regard that as affording any theoretical objection to the proposed definition."¹⁴²

With the problem of belief we shall not be concerned here.¹⁴³ A proposition is either true or false. "Derivatively we call a belief true when it is a belief in a true proposition, and a disbelief true when it is a disbelief in a false proposition; but it is to propositions that the *primary* formal meanings of 'truth' and 'falsehood' apply . . . The attempt to define truth this way seems to be a mistake."¹⁴⁴

Three requisites are met by a proper definition of truth.

¹⁴⁰*Ibid* 500.

¹⁴¹P. of P. 187. What is contended here indicates a possible change of point of view. In M.T. 523-4, for example, he makes the following statement: "It may be said—and this is, I believe, the correct view—that there is no problem at all in truth and falsehood; that some propositions are true and some false, just as some roses are red and some white . . . What is truth and what falsehood, we must merely apprehend, for both seem incapable of analysis."

¹⁴²H.P.M. 43.

¹⁴³This involves propositions of the form "I believe p" which we decided to omit. (See above p. 29) Russell's discussion of this problem is to be found in the following references: P. of P. chap. xii; L. A. 47-63; M.T. *passim*.

¹⁴⁴H.P.M. 42-3.

It "(1) allows truth to have an opposite, namely falsehood, (2) makes truth a property of beliefs, but (3) makes it a property wholly dependent upon the relation of the beliefs to outside things."¹⁴⁵ In view of Russell's later position, if we substitute for "belief" the word "proposition," we shall have the problem more accurately stated. No attempt is made by Russell to justify these necessary characteristics of truth; they are taken as self-evident.

The resulting definition of truth is that "which has on the whole been commonest among philosophers—that truth consists in some form of correspondence between belief [proposition] and fact."¹⁴⁶ More accurately stated, a proposition is "*true* when it corresponds to a certain associated complex, and *false* when it does not . . . This constitutes the definition of truth and falsehood."¹⁴⁷ The complex is called the "fact corresponding to the belief [proposition]."¹⁴⁸ This is essentially the view expressed by Russell in his other writings and may serve as a starting point for our discussion. There are three questions: (1). What is meant by a proposition? (2) What is a fact? (3). What sort of a relation is "correspondence"?

(1). We have considered this question in Chapter Three. One point only needs to be emphasized here. Propositions on the verbal side "are always themselves positive facts."¹⁴⁹ Propositions-verbal may differ by the absence or presence of the word "not," and the meaning may therefore be positive or negative, but the proposition as a form of written or spoken words is a "fact in exactly the same sense in which its objective is a fact. The relation of a proposition to its objective is not a relation of something imagined to something actual; it is a relation between two equally solid and equally actual facts."¹⁵⁰ The problem of truth, then, is one of determining the possible relations which may subsist between two such facts.

(2). It is taken as a truism that "the world contains facts which are what they are whatever we may choose to think about them."¹⁵¹ Facts "belong to the objective world and . . . are not created by our thoughts or beliefs."¹⁵² They are those things which are required in "a complete description of the existing world."¹⁵³ Such a description "would require not only a catalogue of the things, but also a mention of their qualities and relations." Thus "there are facts about the properties of single things; and facts about the relations between two things,

¹⁴⁵It is apparent that the third requisite excludes the coherence theory entirely. (P. of P. 193.)

¹⁴⁶P. of P. 190.

¹⁴⁷*Ibid.* 201.

¹⁴⁸*Ibid.* 202.

¹⁴⁹H.P.M. 39.

¹⁵⁰*Ibid.* 37.

¹⁵¹L.A. 500.

¹⁵²*Ibid.* 502.

¹⁵³S.M. 51.

three things, and so on."¹⁵⁴ But there is no dualism of true and false facts. Certain facts, i.e. propositions, are classed as true or false, but facts as a whole "are only just facts."¹⁵⁵ They may be positive or negative, as we shall see, but to say that a fact is true implies that it may be false, which is obviously impossible.

A fact "is anything complex" by which is meant anything which has "constituents."¹⁵⁶ It is "expressed by a whole sentence, not by a single name like 'Socrates'."¹⁵⁷ That which is designated by the word "Socrates" is a complex in that it is made up of a class of classes and series of appearances. But such a complex is not a fact. A fact is not a particular existing thing like the rain or sun. The complexity of a fact is shown by the circumstance that its constituents may occur as elements in other facts. A particular "this," for example, may be a part of the fact "this is red" and also of the fact "this is before that." This is the sense in which facts can be analyzed.

(3). The relation which a true proposition has to the fact expressed by it is called variously "correspondence,"¹⁵⁸ "reference,"¹⁵⁹ "pointing toward,"¹⁶⁰ "mirroring,"¹⁶¹ "parallelising,"¹⁶² and "symbolizing."¹⁶³ These are evidently to be taken as equivalents. Most of them suggest the idea of resemblance, and in certain simple cases this is a permissible interpretation of the relation. In such a proposition as "Socrates precedes Plato" for example, "the objective which verifies it results from replacing the word 'Socrates' by Socrates, the word 'Plato' by Plato, and the word 'precedes' by the relation of preceding between Socrates and Plato. If the result of this process is a fact, the proposition is true."¹⁶⁴ It is clear that in such a case the proposition actually mirrors the fact and the resemblance is a matter of form. Yet this implication of resemblance gives rise to difficulties with regard to false propositions which "refer" to facts and yet do not "resemble" them. For this reason the words "reference" and "pointing toward" have the advantage.

It is important to note that the relation of a proposition to a fact cannot be called "naming." We can name only objects which are actually present: there is no negative naming relation. Hence propositions could not be said to name facts for that would demand that every proposition be true, i.e. that

¹⁵⁴L.A. 503.

¹⁵⁵*Ibid* 504.

¹⁵⁶H.P.M. 1.

¹⁵⁷L.A. 501.

¹⁵⁸P. of P. *loc. cit.* also L.A. 38-9.

¹⁵⁹H.P.M. *passim*

¹⁶⁰A.M. 273.

¹⁶¹L.A. 520.

¹⁶²H.P.M. 40.

¹⁶³L.A. 504.

¹⁶⁴A.M. 278.

it refer to a fact. But if a proposition were false, it would be naming a non-existent which is obviously impossible. The relation of a proposition to a fact must be such as to permit a negative.

It might seem, then, that the problem would take the following form: If a proposition is true, there is a fact to which it refers; if it is false, there is no such fact. Hence a false proposition has a relation to a non-existent. But this is evidently not the case. Just as a true proposition gets its truth by relation to a fact, so a false proposition must obtain its falsehood by virtue of a relation to a fact. And the fact in this case is precisely the same fact that makes the contradictory proposition true. Hence a proposition and its contradictory both refer to the same fact; the difference is in the *nature* of the relation. A true proposition "points toward" a fact, the corresponding false proposition "points away" from the same fact. In very simple cases "we can say that true propositions actually resemble their objectives in a way in which false propositions do not."¹⁶⁵

Consider an example. The proposition "Socrates is mortal" is a complex symbol predicating mortality of Socrates. Suppose this to be a true proposition. Then there is a fact "Socrates is mortal," and the proposition can be said to mirror the fact. Take the proposition "Socrates is not mortal." This is a false proposition, but its falsehood is determined not by "mirroring a non-existent fact," but by "not-mirroring an existent fact," namely the fact "Socrates is mortal." The relation of "not-mirroring" is not the "indefinite negative" but a definite and ascertainable relation. Every proposition, therefore, refers to a fact, though the reference may be either direct or indirect.

This implies, of course, that there are negative facts. For the truth of such a proposition as "Socrates is not alive" is due to a direct reference to a fact; this fact must be "Socrates is not alive." And the proposition "Socrates is alive" will derive its falsehood from an indirect reference to the fact "Socrates is not alive." Thus there are two forms of facts, just as there are two forms of propositions.¹⁶⁶ But there is "no identity between the distinction of positive and negative facts, and the distinction of positive and negative word-propositions: the latter

¹⁶⁵A. of M. 273.

¹⁶⁶In L.A. 42-6, Russell expresses skepticism as to whether there are negative facts. He examines Mr. Demos' argument (Mind, April 1917) in which he attempts to avoid negative facts by substituting incompatibility. Russell rejects this on the grounds that it is just as difficult to assume "incompatible facts" as "negative facts." In H.P.M. 5, he argues further that we cannot substitute absence of fact for negative fact, because absence of fact is itself a negative fact: "it is a fact that there is no such fact." In this article and in A.M. 275-6, he unhesitatingly asserts the existence of negative facts. It is unfortunate that he has not defined negative facts, or at least told us more about them.

are themselves both positive facts, though differing by the absence or presence of the word 'not'.¹⁶⁷ But one point needs to be emphasized. "It must not be supposed that the negative fact contains a constituent corresponding to the word 'not.' It contains no more constituents than a positive fact of the correlative positive form. The difference between the two forms is ultimate and irreducible."¹⁶⁸ As to the nature of this ultimate and irreducible difference, Russell has told us nothing.

In particular the nature of the reference between propositional-meaning and fact is dependent upon the type of proposition; there is a different kind of reference for each form of proposition. In the atomic proposition, the correspondence is most direct; the fact "is obtained by replacing each word by what it means, the word meaning a relation being replaced by this relation among the meanings of the other words."¹⁶⁹ (Russell is apparently using the word "mean" here in the sense of "objective reference.") In general, then, the atomic proposition is the most accurate "picture" of a fact; the form of the proposition is identical with the form of the fact. True atomic propositions correspond more directly than false, for the reference of a false proposition to a fact is indirect. The fact expressed by the atomic proposition is called by analogy the "atomic fact."

The simplest type of atomic judgment, we have seen,¹⁷⁰ is the judgment of perception,¹⁷¹ such as "this is to the left of that." This is a form of knowledge by acquaintance, and as such "is only possible when there really is such a fact."¹⁷² It would seem impossible, as a consequence, to have a false judgment of this type; every judgment of perception must be true. This is not the case. The possibility of fallacy occurs in the passage from the perception to the judgment.¹⁷³ Every fact is complex, though it is not perceived as complex; the complexity is only apparent upon analysis. But the transition from perception to judgment is precisely this analytic process. Here it is possible for fallacy to creep in. The object of perception would be indicated by "this-is-to-the-left-of-that"—an unanalyzed whole; the object of the judgment is "this is to the left of that." But the judgment is true only when there is a fact corresponding to the complex as analyzed; it is evident that such may be the case. A judgment of perception may therefore be false.

¹⁶⁷H.P.M. 39.

¹⁶⁸*Ibid* 4.

¹⁶⁹A.M. 278. Image-propositions have even a more perfect correspondence. See pages 273-4.

¹⁷⁰Above p. 30.

¹⁷¹The word "perception" is used "in a large sense" here. See P. of P. 211.

¹⁷²P. of P. 212.

¹⁷³*Ibid*. 214.

The way in which molecular propositions refer to fact is noticeably different. With regard to atomic propositions "there is just one fact which is pointed to by the proposition, pointed to either truly or falsely . . . But when you take such a proposition as 'p or q' . . . there you will have two different facts involved in the truth or falsehood of your proposition . . . There will be the fact that corresponds to p, and there will be the fact that corresponds to q, and both of these will be relevant in discovering the truth or falsehood of 'p or q'."¹⁷⁴ The truth of such a proposition, therefore, depends upon two facts, not one. This frees us from the necessity for supposing that there is a disjunctive fact "p or q"; "we must not look about the real world for an object which we can call 'or' and say, 'Now look at this. This is "or"'"¹⁵⁷ And the same must apply to such words and expressions as "if-then," "and," "not-both," etc. i.e. there are no molecular facts.

Russell devotes very little space to the problem of the existence of molecular facts, and expresses frank skepticism about the whole matter.¹⁷⁶ It seems to me that however unpleasant the notion may be to us, we must assume the existence of molecular facts. This is especially evident with regard to the conjunctive proposition "a and b," which Russell includes among the molecular propositions.¹⁷⁷ Let us consider such a proposition, supposing for the moment that a and b are atomic propositions. The proposition "a and b" is molecular and therefore differs from the atomic propositions which make it up; it consists, that is, of the two atomic propositions together with the notion "and." The notion "and" is supposed to be something over and above atomic propositions. According to Russell's view, the molecular proposition "a and b" depends for its truth upon the existence of the two atomic facts "a" and "b". But if the molecular proposition which says that the two propositions are both true, is an advance in knowledge over the two statements "a is true" and "b is true," then certainly "a and b both exist" must be a fact over and above the two facts "a exists" and "b exists." If our parallelism of proposition and fact is to hold throughout, the difference between atomic and molecular propositions must exhibit itself in a corresponding difference between atomic fact and some other fact which consists of an atomic fact plus something. In the interest of economy we should call this "atomic fact plus something" a molecular fact.

Consider what would be the case if such a proposition were false. The contradictory is "Either not-a or not-b," and if the original proposition is to be false, this must be true. But a dis-

¹⁷⁴L.A. 38-9.

¹⁷⁵*Ibid.* 40.

¹⁷⁶L.A. 46.

¹⁷⁷*Ibid.* 37.

junctive proposition is equivalent to an implicative relation; hence we may express the proposition "not-a or not-b" as "a implies not-b" or in its more common form, "if a, then not-b."¹⁷⁸ If there are molecular facts, then the truth of this hypothetical proposition will depend somehow upon the existence of facts a, not-a, b or not b. Evidently a and not-a cannot both exist, nor can b and not-b, yet either a or not-a and either b or not-b must exist. Consequently we have four possibilities: a and not-b, not-a and not-b, not-a and b, a and b; in the first three the proposition will be true, in the last false. Now let us ask the question: Upon the existence of what atomic fact does the truth of such a proposition depend? The answer seems to be the following: Whenever not-a exists, the proposition is true. But this is only a sufficient condition, not a necessary one; we cannot convert and say: whenever the proposition is true, not-a exists. Consequently we cannot take the existence of not-a as a *defining* characteristic of the truth of the proposition "a implies not-b." It must be added that the existence of a in conjunction with not-b is also a sufficient condition for determining the truth of the given proposition. The truth of "if a, then not-b" is therefore defined as: "Either not-a or else 'a and not-b'." But this attempt to avoid molecular facts leads only to a reappearance of the problem in a different form; the truth of a molecular proposition depends upon the truth of another molecular proposition.¹⁷⁹ If we are to avoid an infinite regress, we must assume molecular facts.

If there are no molecular facts, then there is no factual basis for reasoning. All reasoning depends upon those relations between atomic propositions which are expressed by molecular propositions. The assumption of molecular facts involves supposing that there are actual relations existing between atomic facts corresponding to the notions "if-then," "or" and the rest. That means that we shall "have to look about the world for an object which we can call 'or'."¹⁸⁰ Personally I can see no reason why one should object to this. A relation called "or" is certainly as tangible and as much demanded by the nature of things as a relation called "before." This, however, is a very large question which the logical atomism of the future will have to answer.

When we reach the final type, general-particular propositions, the parallelism between propositional-meaning and fact is once more complete. In "the inventory of the world . . . in

¹⁷⁸Russell suggests (L.A. 200) that the interpretation of this sort of proposition leads into difficulty if one denies molecular facts. He does not state or explain the difficulty.

¹⁷⁹As a matter of fact instead of simplifying our problem we have complicated it, for the definition of the truth of the implicative proposition involves a molecular proposition which itself contains a molecular proposition.

¹⁸⁰As referred to above p. 53.

addition to particular facts . . . there are also general facts and existence facts, that is to say, there are not merely *propositions* of that sort but also *facts* of that sort."¹⁸¹ The grounds for assuming general facts are as follows: Such a proposition as "All men are mortal" cannot be known by inference from atomic facts alone. If we could know each individual man and know that he was mortal, that would not enable us to know that all men are mortal unless we *knew* that these were all the men, which is a general proposition . . . Thus general truths cannot be inferred from particular truths alone, but must, if they are to be known, be either self-evident, or inferred from premises of which at least one is a general truth."¹⁸² It is strange that Russell should admit general facts when he has denied molecular facts, since the grounds in favor of the former are the same in character as those which we advanced in favor of the latter. Since general propositions cannot be inferred simply from atomic propositions, they evidently contain notions which are not contained in atomic propositions. Conspicuously among these is the idea of "all." Consequently when we come to define the truth of a general proposition, it cannot be by reference simply to atomic facts, for no combination of atomic facts will give the notion "all." It must be a further fact that "these are all the facts." But this is a different type of fact, and can be included neither among the atomic facts, nor among the molecular facts if there are any.

The same considerations apply to particular facts, though not so obviously. Existence facts are those which are "separate and distinct facts over and above the atomic facts . . . those facts have got to come into the inventory of the world."¹⁸³ There are therefore such facts as "some men are mortal" and such a fact is different in type from the facts "John is mortal," "Henry is mortal," etc. That is, we have added the notion of "some." But this gives rise to two difficulties which Russell fails to consider. The first is that the notion "some" is defined to include "one." It follows from this that in certain cases the truth of a particular proposition is determined by the existence of an atomic fact. For example the proposition "Some men are now president of the United States" is a true proposition; but its truth is determined by virtue of a direct reference to the fact "Coolidge is now president of the United States," which is an atomic fact. The second difficulty lies in considering the proposition "There are men." This is an existence-proposition or particular proposition, and should derive its truth by direct reference to a particular fact, "There are men." But it is hard to see how this fact differs at all from the molecular fact "John is a man and Henry is a man and

¹⁸¹L.A. 198.

¹⁸²S.M. 55-6.

¹⁸³L.A. 200.

James is a man . . .” The particular proposition is obviously true if this molecular fact exists, and the truth of a particular proposition is therefore *sometimes* determined by a molecular fact. The only alternative is to show that the notions “there are” or “some are” cannot be inferred from “x and y and z are,” without the addition of some further idea. It is difficult to conceive what this idea could be.

The conclusion of this chapter concerning the existence of types of facts must be to express a frank skepticism about the whole matter—a skepticism, however, which is only tentative and therefore not agnosticism. The *means* for determining the solution are evidently perfectly concise once the analysis of propositions has been properly carried out. But there are obvious difficulties, some of which have been indicated. Russell asserts the existence of atomic and of general and particular facts, but denies the existence of molecular facts. We have presented certain considerations which demand molecular facts, and certain others which make the existence of general and particular facts at least doubtful. The final conclusion therefore depends upon the accumulation of further evidence. Russell deserves credit for having realized the problem, and for having offered the premises for its solution.

If we accept Russell’s results, then several volumes must be added to the catalogue of the world. Beyond the realm of particulars, yet not within the world of universals, there must exist a world of facts, which differs from each of the other spheres in essential characteristics. Within this realm of facts there is an infinite hierarchy of facts, on each level of which (due to the doctrine of types as applied to propositions) there is an infinite hierarchy of facts. Of this infinity of infinities we have seen only two instances, the atomic facts and the general and particular facts. Russell has suggested at least two other types of propositions, those involving believing, and those asserting the absolute generalities of logic. It is possible that there are also facts of these sorts. At any rate we can never completely list the different types of facts.

Russell considers it advisable to stop here. He says that the *only* other sort of object (beyond particulars and universals) which we come across in the world is the fact. Unless complexes of facts are also facts, it is pure dogmatism to stop at this point. The proposition is only the unit of thought; this suggests that there are also complexes of thought. A very simple example of such a complex would be: A believes that b is a man and all men are mortal. Keyser¹⁸⁴ has considered it necessary to create a “doctrinal function” which is a form for relations of propositions, just as Russell’s “propositional-function” is a form for relations of terms. The truth of a “doctrine” would

¹⁸⁴*Mathematical Philosophy.*

seem to lie in a direct reference to a "complex of facts," which should be called a new type of existent.

It seems strange that Russell should have considered the necessity of apologizing "for any sort of dryness or dullness in the world."¹⁸⁵ One is inclined to wonder and speculate as to what sort of *Weltanschauung* he would have constructed had he not worshipped "Occam's razor."

¹⁸⁵L.A. 380.

CHAPTER FIVE

GENERAL CONCLUSIONS

Since our treatment has been for the greater part uncritical, it will be well to add a few words in conclusion in order to indicate what in Russell's presentation must be rejected, what must be accepted and consequently the direction in which to look for the complete solution of the problem of Logical Atomism.

Russell's conclusions are, on the whole, correct. When certain necessary distinctions have been made with regard to the nature of language as a symbol, and when the assumption involved in the whole procedure has been explicitly stated and granted, the deductions which Russell makes are a necessary consequence. Assuming the identity of form of proposition and form of fact expressed by the proposition, it is possible by empirically enumerating forms and types of propositions to conclude that there is a corresponding hierarchy of facts, each type of which is irreducible to any other type. Consequently in describing the world we must enumerate not only its *elements*, but the different arrangements of the elements, i.e. the *complexes* as well. It is not sufficient to know simply that the world contains this, that and the other particular; we must determine how particulars are related, how universals are related to particulars, how universals are related among themselves (if at all), and possibly even how such complexes of particulars, predicates and relations are related to one another.

The main difficulty, which must make skepticism the ultimate conclusion of a logical atomism, lies in the impossibility of proving the assumption of the identity in form of proposition and fact. This assumption is necessary in order to permit any metaphysical conclusions to be drawn, yet the only basis for the assumption itself is "self-evidence." The reason for this is the fact that form can never be described or talked about without begging the question. To talk about the form of a proposition or of a fact is to assert a proposition about this form; but such a proposition will itself have a form, and it becomes necessary to explain *this* form. Consequently, since structure cannot be described, it is never possible to prove the identity in form existing between proposition and fact. Proposition and fact "show forth" the same form, but to one who cannot "see" this identity, it must forever remain a mystery why such an assumption should be considered self-evident. The resort to self-evidence is always unwarranted as it is an appeal to psychological rather than to logical grounds.

A further danger is involved in the general problem of symbolism. Russell does not entirely avoid this pitfall himself. He states explicitly that "unless you are fairly self-conscious about symbols, unless you are fairly aware of the relation of the

symbol to what it symbolizes, you will find yourself attributing to the thing properties which belong only to the symbol."¹⁸⁶ We must not assume that "the structure of a sentence corresponds at all accurately with the structure of the fact that it asserts."¹⁸⁷ The danger in assuming such an identity is in making "grammar the key to metaphysics" and constructing a linguistic rather than a rationalistic interpretation of the world. Identity of grammatical construction in two given cases, even though it be combined with identity of verbal symbol, is not a sufficient warrant for concluding identity in structure for the corresponding facts. For example the verbs in the two cases "Socrates is human" and "Socrates is a man," though indicated by the same word and occupying the same grammatical construction, symbolize two different relations: the former expresses predication, the latter symbolizes identity between an object named and an object ambiguously described.¹⁸⁸ But these are essentially different relations and the facts of which they are constituents must differ fundamentally.

The analysis of word-meanings which I have considered in Chapter Two is probably not entirely correct, though it seems likely that the division of terms into proper names, general names and relational words affords a satisfactory starting-point. This classification seems to be based upon a natural division of thought, and not merely upon function in a sentence; it is consequently logical and not simply grammatical. Furthermore, it seems necessary to admit that there are such things as "incomplete symbols," especially with regard to descriptive phrases. I think, however, that Russell has not adduced sufficient arguments to establish the fact that general names and verbs are incomplete symbols and have no meaning in isolation. Russell tends to identify the problem of what words *mean* with the problem of what is involved in the *understanding* of a word. These are really two distinct problems, and to solve the former by an appeal to the latter is unwarranted. It is true that in order to understand the universal "blue" we must understand what it is for something to be "blue," i.e. the universal cannot be understood in isolation. But this does not imply that the word "blue" has no meaning apart from the context of a proposition.

An important question in this connection concerns relations. Do relations have instances or are there only "universal relations"? Is the relation of resemblance which A has to B exactly and numerically the same as the relation of C to D, or is each of these an instance of the general relation of resemblance? We touched upon this matter in Chapter Two and saw that Russell inclines toward the former position, though he

¹⁸⁶L.A. 504.

¹⁸⁷A.M. 212.

¹⁸⁸I.M.P. 172.

does not consistently hold to this view. The answer which is given to the question is of great importance since if relations have instances, the universal relation may be *predicated* of the instance, i.e. a relation may be a predicate. . But this gives rise to difficulties, since predicates and relations are of different types.

The general empirical analysis and classification of proposition into atomic, molecular and general-particular is, I think, beyond criticism. There seems to be, as a matter of fact, these different types of proposition which are irreducible to one another. The one important aspect of this matter for traditional logic is the admission of relational propositions which are fundamentally different in structure from S-P propositions. Practically this means that courses in formal logic as taught today must be reconstructed so as to permit greater emphasis on "extra-syllogistic" arguments.

There is still much to be done with general-particular propositions. If our results are true, a sharp distinction must be made between the extensional and the intensional interpretations of such propositions. If Russell's symbolism is to be adopted, it must be applied only to the extensional side, and the intensional interpretation must not be condemned because it fails to accord with this symbolism. In line with this, of course, a further development of the intensional interpretation is demanded.

The answers to the foregoing questions will determine the nature of the metaphysical consequences. If general names and verbs can be shown to have meaning in isolation, it is quite likely that they will have existence in isolation, which implies that their existence will be not inferred but immediate. They will therefore play their part in the actual make-up of the world as elements of its structure. If relations have instances then we must admit that the classification into types applies to particulars as well as to predicates and relations, for there will be particular relations just as there are particular colors. However, even though we admit the logical doctrine of types, it is possible that in passing to types of existence we are "attributing to the thing properties which belong only to the symbol." It seems to me that further grounds are necessary to show that the doctrine of types is metaphysical as well as logical.

It is hard to see on what grounds Russell's theory of truth could be criticised, since it is not to be taken as a *test* of truth. A test is something that may be applied; Russell's theory can never be applied for a fact can never be perceived. We perceive only simples; we analyze many of what were perceived as simples, and find them to be made up of elements, but the analysis may be faulty and we can never be sure that the elements exist in the relations we have ascribed to them. The only theoretical grounds which could be brought against Russell's

theory would be those showing it to be either self-contradictory or else inconsistent with the rest of his system. In itself the theory seems to be adequate. In relation to his other doctrines, it seems impossible to criticise the theory until Russell tells us more definitely what he means by a "fact." As long as fact is defined as "that which makes a proposition true or false" it is impossible to object to the definition of truth as "a reference of a proposition to a fact." Russell has said nothing, to be sure, but what he has said is correct.

With regard to the problem of the existence of types of facts, Russell has merely suggested the solution. What he has to say about atomic facts is probably correct; his conclusions with regard to molecular and general-particular facts are probably incorrect. An interesting and important problem concerning molecular facts lies in the metaphysical equivalents of the symbols "if-then," "therefore," etc.; what is it, that is, which makes a molecular fact different from an atomic? What is there in the real world which corresponds to the logical notion of "implication"? The direction in which the solution to this problem lies is in passing on to the type of proposition which we have omitted—the "absolutely general" propositions of logic. What is there in the real world corresponding to the proposition "If a is greater than b , and b is greater than c , then a is greater than c "? How do propositions of logic "refer" to fact?

It is to these questions that the logical atomism of the future will have to devote its attention. Very little of permanent worth can be done until symbolic logic has reached a higher stage of perfection. With the increased interest in this field of study, it seems likely that this stage is not far off. The pity is that the precise balance of mathematical acumen and philosophical insight which is required for such investigation is so rare. It is because Russell has these two qualities combined in the desirable proportions, that he has been able to contribute so much to the subject. For the fact remains that however greatly one may criticise his position, he deserves the credit of the pioneer. The first step in a new field is always the most difficult to make. Russell has made this initial step, faltering to be sure, but with the courage of conviction that somewhere in the entanglements and underbrush there is to be found firm footing. It must be the task of others, perhaps less sturdy than he, to retrace his steps and clear the path of pitfalls which he, in his haste to explore, avoided either by indirect and roundabout paths or by building rather flimsy, temporary bridges of "logical abstractions."

BIBLIOGRAPHY

The following is a chronological list of Russell's published writings insofar as they pertain either to his logical atomism, or to his general metaphysical position. The contributions to ethics and political philosophy are omitted, as are also the primarily mathematical writings. Articles which appeared in both periodical and book form are included under the latter reference.

<i>Title</i>	<i>Periodical and date</i>	<i>Abbrevi- ation</i>
Essay on the Foundations of Geometry	1897	
Critical Exposition of the Philosophy of Leibniz	1900	P. of L.
On the Notion of Order	Mind 1901	
Is Position in Time and Space Absolute or Relative?	Mind 1901	
On Meinong's Theory of Complexes	Mind 1904	M.T.
Existential Import of Propositions	Mind 1905	
On Denoting	Mind 1905	
Principles of Mathematics	1905	P. of M.
Nature of Truth	Mind 1906	
On the Nature of Truth	Arist. Soc. 1906-07	
Some Explanations in Reply to Mr. Bradley	Mind 1910	
Philosophical Essays	1910	
Principia Mathematica	3 vol. 1910-13	P.M.
The Basis of Realism	Journ. Phil. Psy. and Sci. Meth. 1911	
On the Relation of Universals and Particulars	Arist. Soc. 1911-12	
Problems of Philosophy	1912	P. of P.
Philosophical Importance of Mathematical Logic	Monist 1913	
Nature of Sense-data	Mind 1913	
On the Nature of Acquaintance	Monist 1914	
Definitions and Methodological Principles in Theory of Knowledge	Monist 1914	
Sensation and Imagination	Monist 1915	
On the Experience of Time	Monist 1915	

Ultimate Constituents of Matter	Monist 1915	
Our Knowledge of the External World as a Field for the Scientific Method in Philosophy	1915	S.M.
Mysticism and Logic	1917	M.L.
Philosophy of Logical Atomism	Monist 1918-19	L.A.
Introduction to Mathematical Philosophy	1919	I.M.P.
Professor Dewey's "Essays in Experimental Logic"	Journ. Phil. Psy. and Sci. Meth. 1919	
How Propositions Mean	Arist. Soc. Supp. Vol. II, 1919	H.P.M.
Meaning of Meaning	Mind 1920	
Analysis of Mind	1921	A.M.
Physics and Perception	Mind 1922	
Dr. Schiller's Analysis of "The Analysis of Mind"	Jour. Phil. 1922	

